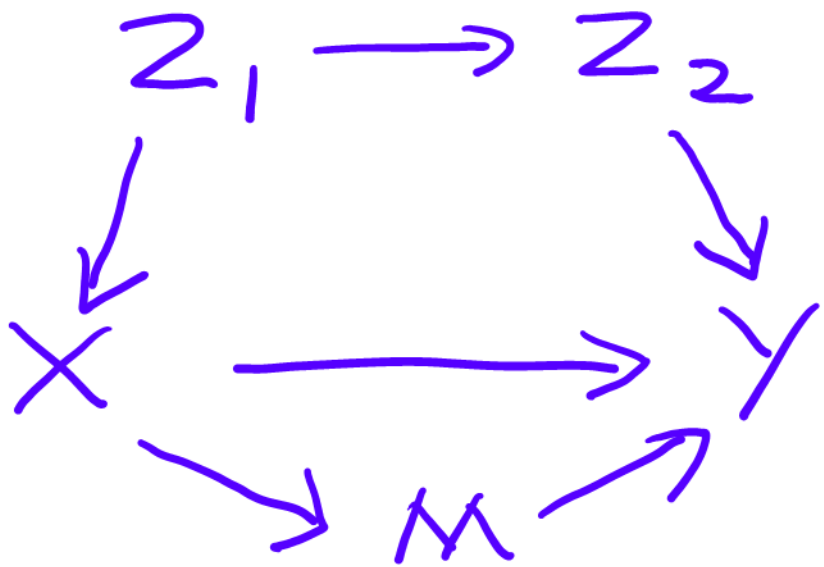
More sample questions for the final exam

1) Sample mid-term and mid-term questions
2) Redo the sample question about a lower triangular factorization $G = AA^T$ with an upper triangular factorization.
3) Suppose a positive definite matrix Σ can be factored as $\Sigma = AA^T = BB^T$ where A and B are square. Show there exists an orthogonal Γ such that $A = B\Gamma$.

4) Consider the output at the end for a non-linear model using nlme. Draw the predicted response curve for the first 1,000 days post-coma for a patient with 0 days in coma and for a patient with 100 days in coma.

5) Consider the following causal graph for the causal relationships among variables. You wish to estimate the causal effect of X on Y.



You may assume linear relationships.

Discuss the relative merits with respect to a) whether $\hat{\beta}_x$ is an unbiased estimate of the causal effect of X, and

b) in where $\hat{\beta}_x$ is unbiased for the causal effect of X, compare $SE(\hat{\beta}_x)$

1) $Y \sim X$
2) $Y \sim X + Z_1$
3) $Y \sim X + Z_1 + Z_2$
4) $Y \sim X + Z_2$
5) $Y \sim X + Z_1 + M$

NLME CODE AND OUTPUT:

changing `dayspc` to `dayspc - 30` in the non-linear formula. If this works we can do the same for PIQ to have comparable results.

```
> fit.nlme.viq2 <-nlme( viq ~ b0 + b1*exp(-a*(dayspc-30)),
+          data = iq,
+          fixed = list( b0 ~ 1 + sqrt(dcoma) ,
+                        b1 ~ 1 + sqrt(dcoma) ,
+                        a ~ 1),
+          random = list( id  = list( b0 ~ 1, b1~ 1 )),
+          start = list(
+          fixed = c(100, -.3, -10, 0,.3)),
+          control = list( maxIter = 100, returnObject = T),
+          verbose = T,
+          subset = dcoma < 100)
```

which converges in 3 iterations:

. . . . . .

```
**Iteration 3
LME step: Loglik: -1225.668 , nlm iterations: 1
reStruct  parameters:
       id1        id2        id3
-0.8286352 12.2429072 59.7420069

PNLS step: RSS =  9725.302
 fixed effects:99.2084  -0.561859  -6.79895  -1.87309
0.0214789
 iterations: 7

Convergence:
       fixed     reStruct
1.054542e-07 1.588532e-01
>
> summary( fit.nlme.viq2 )
Nonlinear mixed-effects model fit by maximum
likelihood
```

```
  Model: viq ~ b0 + b1 * exp(-a * (dayspc - 30))
 Data: iq
  Subset: dcoma < 100
       AIC      BIC    logLik
  2469.336 2503.363 -1225.668

Random effects:
 Formula: list(b0 ~ 1, b1 ~ 1)
 Level: id
 Structure: General positive-definite, Log-Cholesky
parametrization
                 StdDev       Corr
b0.(Intercept) 1.254731e+01 b0.(I)
b1.(Intercept) 2.640292e-05 0
Residual       5.478719e+00
```

```
Fixed effects: list(b0 ~ 1 + sqrt(dcoma), b1 ~ 1 +
sqrt(dcoma), a ~ 1)
                    Value Std.Error  DF  t-value p-value
b0.(Intercept) 99.20845 1.7262297 196 57.47117  0.0000
b0.sqrt(dcoma) -0.56186 0.4940133 123 -1.13734  0.2576
b1.(Intercept) -6.79895 2.2442154 123 -3.02954  0.0030
b1.sqrt(dcoma) -1.87309 0.7804948 123 -2.39987  0.0179
a               0.02148 0.0044938 123  4.77971  0.0000


 Correlation:
                b0.(I) b0.s() b1.(I) b1.s()
b0.sqrt(dcoma) -0.777
b1.(Intercept) -0.418  0.332
b1.sqrt(dcoma)  0.312 -0.343 -0.860
a              -0.148 -0.057  0.096 -0.058

Standardized Within-Group Residuals:
         Min           Q1          Med           Q3          Max
-2.157276598 -0.389534045  0.007514936  0.366479340  1.903012142

Number of Observations: 324
```