which would be equal if any (assume a very large n and ignore the effect of slight differences in degrees of freedom for the error term). Explain your reasoning briefly.

1. `Y ~ X + G`
2. `Y ~ X`
3. `Yr ~ Xr` where `Yr` is the residual of `Y` regressed on `G` and similarly for `Xr`
4. `Y ~ Xr`
5. `Y ~ X + Xh` where `Xh` is the least-squares predictor of `X` based on `G`
6. `Y ~ X + Xh + Zg` where `Zg` is a `G`-level numerical variable, i.e. it has the same value for all observations with a common value of `G`.

AVP

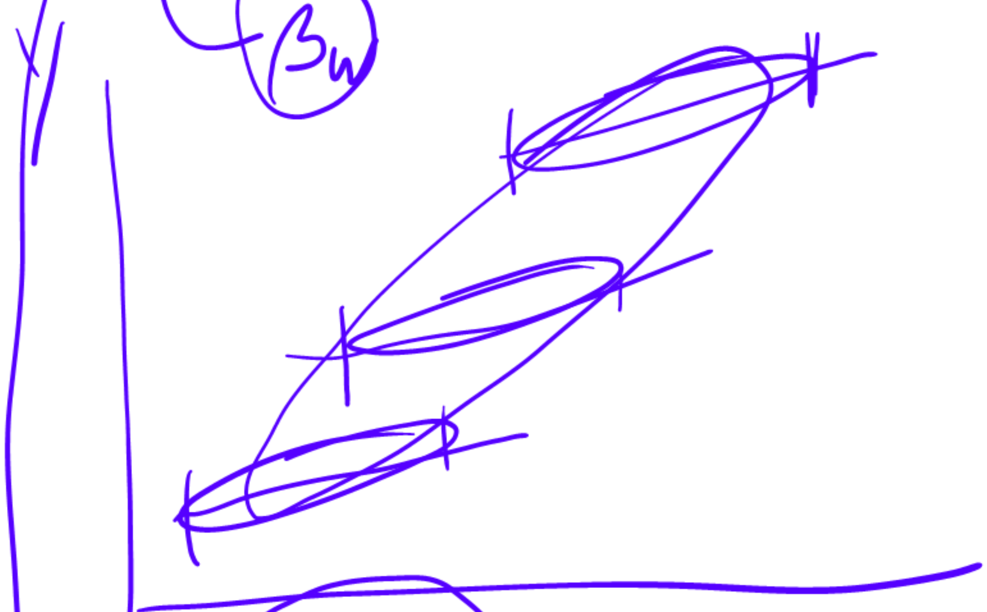
$\hat\beta_W$ $\hat\beta_P$

$X_h \in \mathcal{L}(G)$

$-\hat\beta_W$ X_r

$Var(Y|X_h) \le Var(Y)$

$\hat\beta_W$



$\hat\beta_W$	$\hat\beta_P$
1, 3 (=)	2
5	
4, 6	

SE

$SSE_3 = SSE_1$

$SSE_5 \ge SSE_1$

$SSE_6 \le SSE_5$

$SSE_4 \ge SSE_5$

$SSE_6 \ge SSE_1$

resid $(Y|X_h)$ - resid of X on X_h $\quad \hat X_G \quad X - \hat X_G = X_r$

$var(Y|X_h) \ge var(Y|G) \quad SSE_5 \ge SSE_3$

$$A = \Gamma \Lambda \Gamma'$$

$$A = \Gamma^* \Lambda \Gamma^{*\prime}$$

$$\Lambda = \begin{bmatrix} \lambda_1 & & 0 \\ & \ddots & \\ 0 & & \lambda_p \end{bmatrix} \quad \lambda_i \neq \lambda_j \quad i \neq j$$

$$\gamma_i^* = \pm \gamma_i$$

$$A \gamma_i = \lambda_i \gamma_i$$

$$\Gamma \Lambda \Gamma' \gamma_i = \Gamma \Lambda \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} = \Gamma \begin{pmatrix} 0 \\ 0 \\ \lambda_i \\ 0 \\ 0 \end{pmatrix} = \gamma_i$$

$$A \gamma_i^* = \lambda_i \gamma_i^*$$

$$(A - \lambda_i I) \gamma_i = 0$$

$$(A - \lambda_i I) \gamma_i^* = 0$$

$$\gamma_i, \gamma_i^* \in \ker(A - \lambda_i I)$$

$$A - \lambda_i I = \Gamma \Lambda \Gamma' - \lambda_i \Gamma \Gamma'$$

$$= \Gamma \underbrace{(\Lambda - \lambda_i)}_{\text{one } 0} \Gamma'$$

nh ker$(A) = 1$

γ_i & γ_i^* in same space of dim 1

$\therefore \ \gamma_i = \pm \gamma_i^*$

EX 5 Equiv.

a) A is a var $n \times n$

b) A has SpD with $\min(\lambda_i) \geq 0$

c) A is NND

d) $\exists$ a $B \ni A = BB^T$

Pf: a) $\exists$ rv $\underset{\sim}{X} \ni A = Var(\underset{\sim}{X})$

(a) $\Rightarrow$ (c) Let $\underset{\sim}{c} \in \mathbb{R}^n$

then $0 \leq Var(\underset{\sim}{c}'\underset{\sim}{X}) = \underset{\sim}{c}' A \underset{\sim}{c}$

$\therefore$ A is NND

(c) $\Rightarrow$ (b) $A = \Gamma \Lambda \Gamma'$

$\gamma_n' \Gamma \Lambda \Gamma' \gamma_n = (0\,0\,0 \cdots 1) \Lambda \begin{pmatrix} 0 \\ 0 \\ 0 \\ \vdots \end{pmatrix} = \lambda_n$

Since A is NND $\therefore \lambda_n \geq 0$

$(b) \Rightarrow (d)$ $A = \Gamma \Lambda \Gamma'$ where $\lambda_i \geq 0$

$\therefore$ $\Lambda = \Lambda^{1/2} \Lambda^{1/2}$

Let $B = \Gamma \Lambda^{1/2}$ and $BB^T = \Gamma \Lambda^{1/2} \Lambda^{1/2} \Gamma'$

$= \Gamma \Lambda \Gamma' = A.$

$(d) \Rightarrow (a)$ Let $\underset{\sim}{Z} \sim N(0, I)$

Let $X = BZ$

$\mathrm{Var}(X) = B \underline{\mathrm{Var}(Z)} B^T = BB^T = A$

<u>EX 7</u> $(x'y)^2 \leq (x'Ax)(y'A^{-1}y)$ is A is PD.

Let $A = BB^T$ B^{-1} exists.

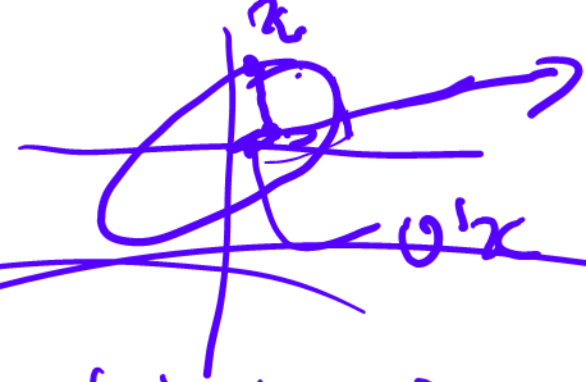
$(x'y)^2 = (x'BB^{-1}y)^2 = ((\underline{B^Tx})' \underline{B^{-1}y})^2$

$$= (x' B B^T x)(y' B^{-1T} B^{-1} y)$$

$$= (x' A x)(y' \cancel{B} A^{-1} y)$$

Ex 8 $\quad \mathcal{E} = \{ \underline{x} : \underline{x}' A^{-1} \underline{x} = 1 \}$

Let $x \in \mathcal{E}$



$$(u'x)^2 \le (u'Au)(\underbrace{x'A^{-1}x}_{1})$$

$$\le u'Au$$

with equality if $x = kAu$

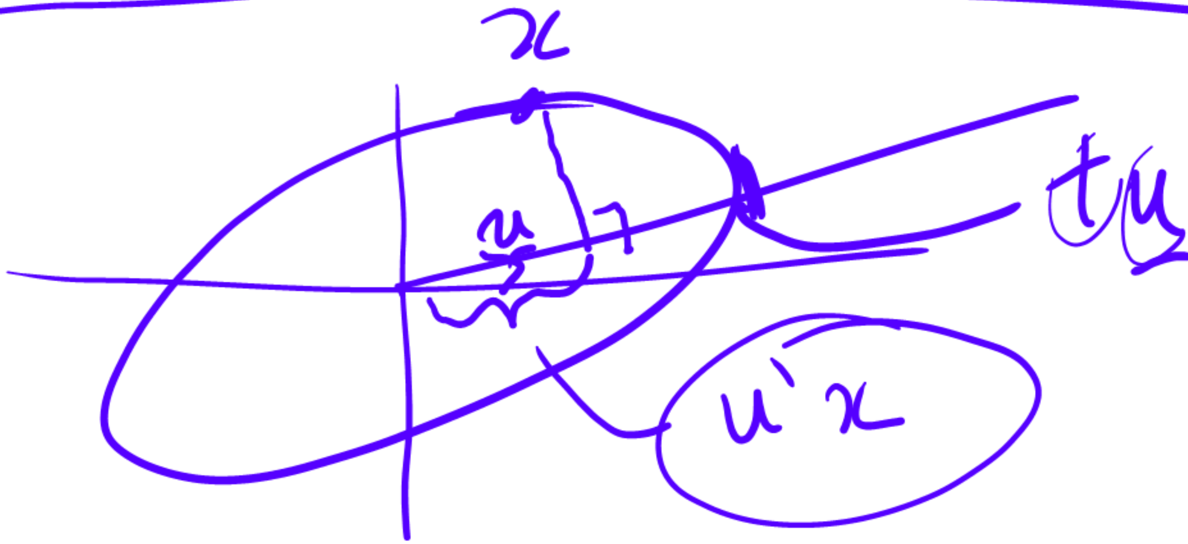
$$k: 1 = x'A^{-1}x = k u' \cancel{A}\, \cancel{A^{-1}} A k u$$

$$= k^2 u'Au$$

$$k^2 = \frac{1}{u'Au}$$

not nec

$\max |u'x| = \sqrt{u'Au}$ max length



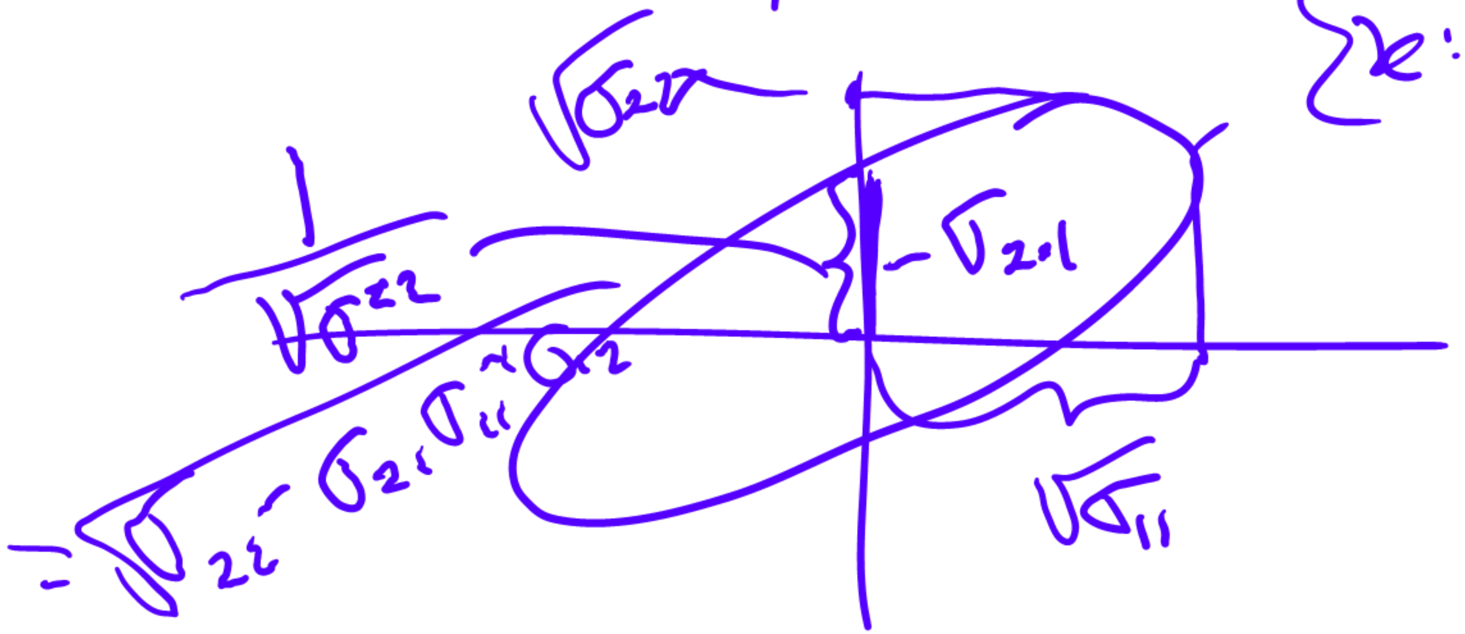
Find: $\underset{\sim}{x} = t\underset{\sim}{u} \ni \underset{\sim}{x} \in \mathcal{E}$

$$t u' A^{-1} t u = 1$$

$$t^2 = \frac{1}{u'A^{-1}u} \qquad t = \frac{1}{\sqrt{u'A^{-1}u}}$$

Let $v = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\quad u = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$\{x : x' \Sigma^{-1} x = 1\}$



$u = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$v = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

A WTS that $B = A^{-1}$
sq.

$BA = \cdots\cdots = I$

$\therefore B = A^{-1}$

$$(A+B)^{-1} = A^{-1} - A^{-1}(B^{-1}+A^{-1})^{-1}A^{-1}$$

$$(A+UDV)^{-1} = A^{-1} - A^{-1}U(D^{-1}+VD^{-1}U)^{-1}VA^{-1}$$

$D = B \quad U = V = I$

$$(A+xx')^{-1} = A^{-1} - A^{-1}x(1+x'A^{-1}x)^{-1}x'A^{-1}$$

$A = X'X$

$$\underset{m\times n}{A} = \underset{m\times r}{U} D \underset{r\times n}{V^T} \qquad r = \operatorname{rank}(A)$$

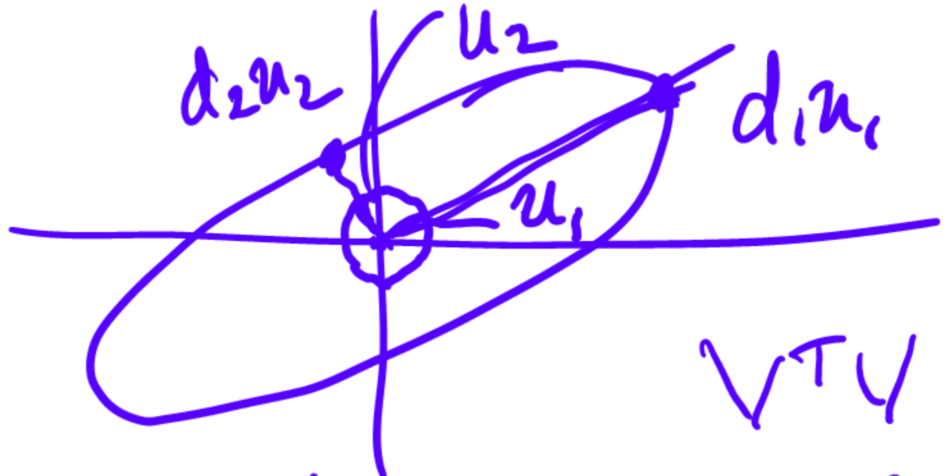
$$\mathcal{L}(A) = \mathcal{L}(U)$$

$$\ker(A) = \mathcal{L}^{\perp}(V)$$

Let $\mathcal{U}$ = unit sphere in $\mathbb{R}^n$

$$\mathcal{U} = \{u : u'u = 1\}.$$

$$\mathcal{E} = A\mathcal{U}$$

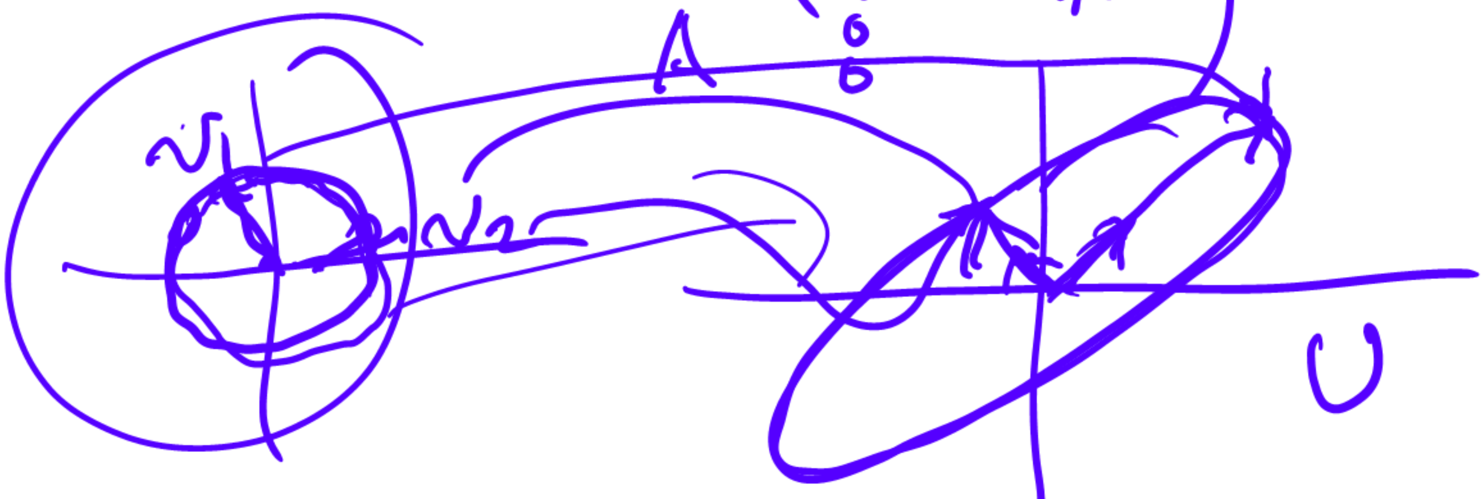


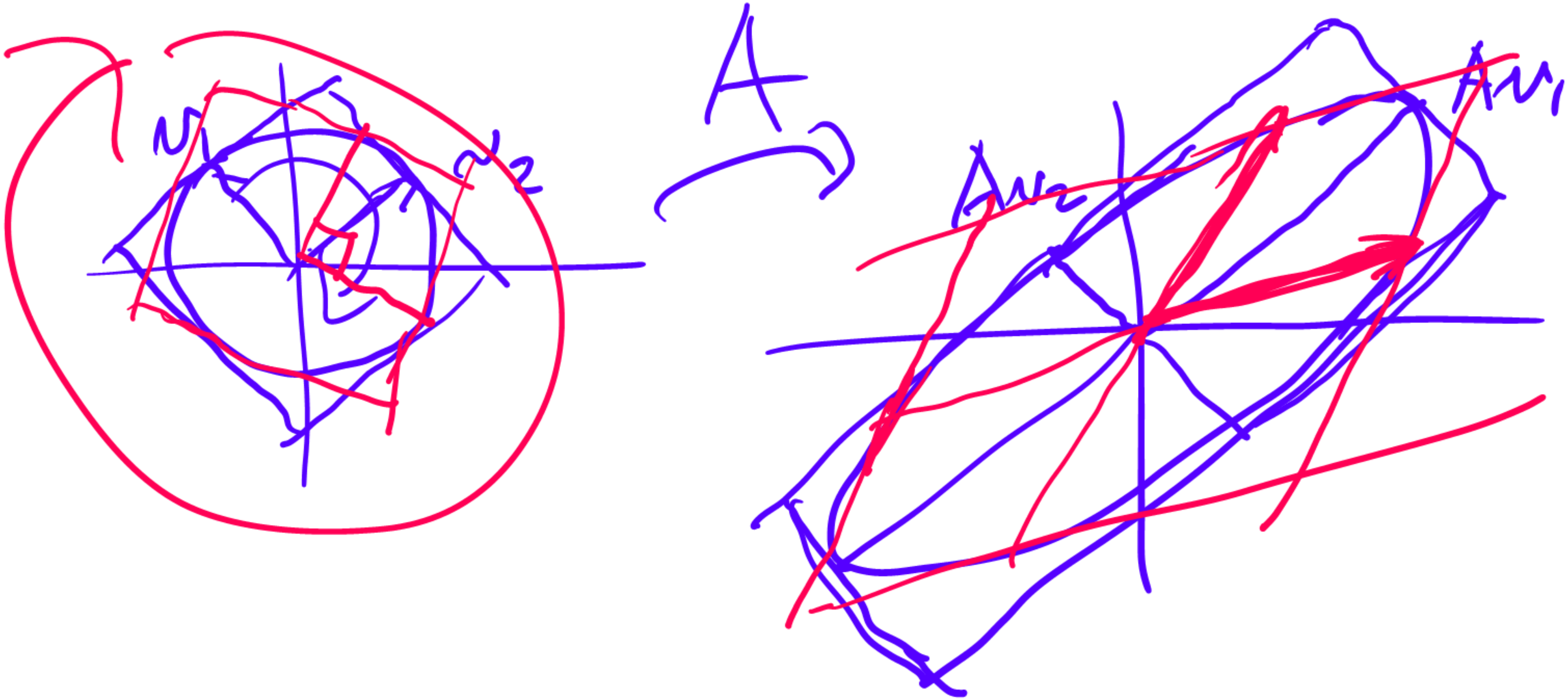
$V^T V$

$d_i u_i = \underline{A}\, v_i$ $\qquad$ v_i is pre image of $d_i u_i$ under A

$$= \underline{U D}\, \underline{V^T v_i}$$

$$= U D \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \text{- ith pos} = U \begin{pmatrix} 0 \\ 0 \\ d_i \\ 0 \\ 0 \end{pmatrix} = d_i u_i$$




v_1
v_2
A
Av_2
Av_1