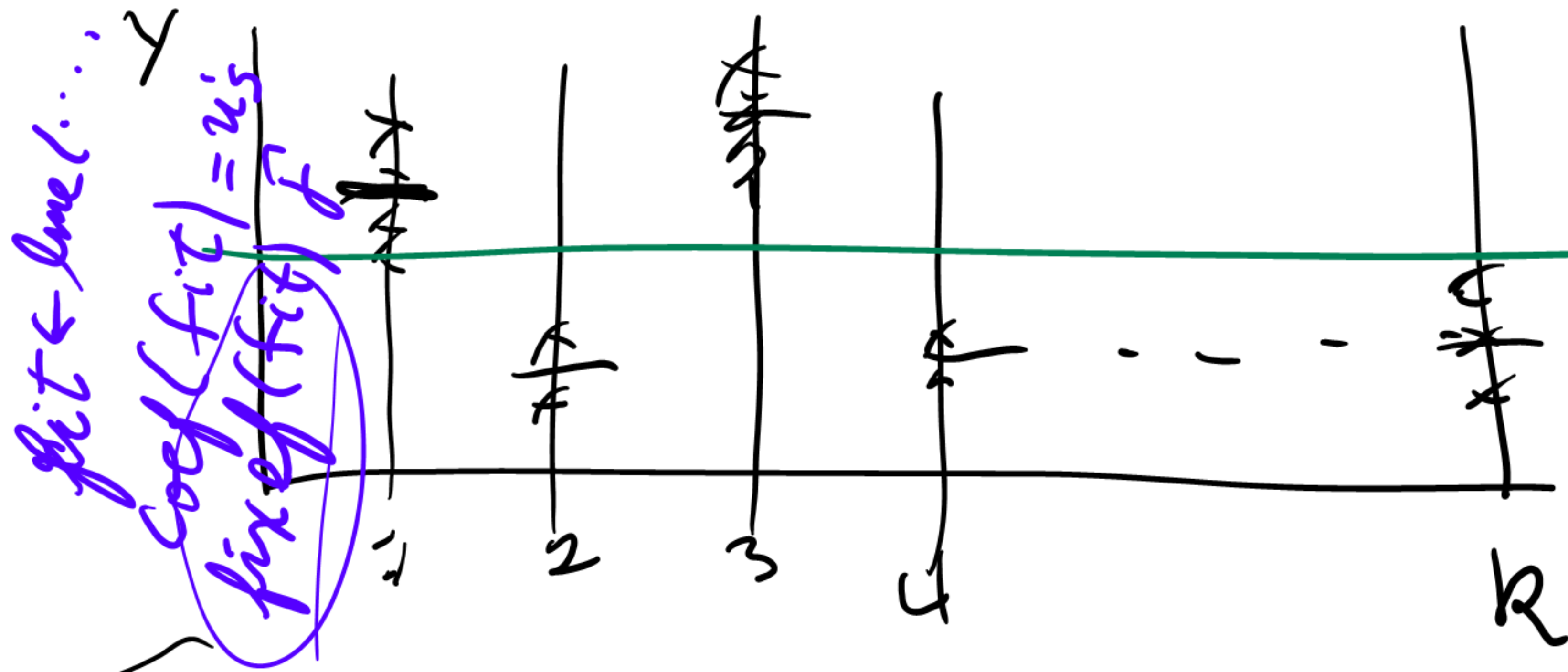


city mean?

$$Y_{ij} = \gamma + u_i + \varepsilon_{ij}$$

school student

Fixed para.

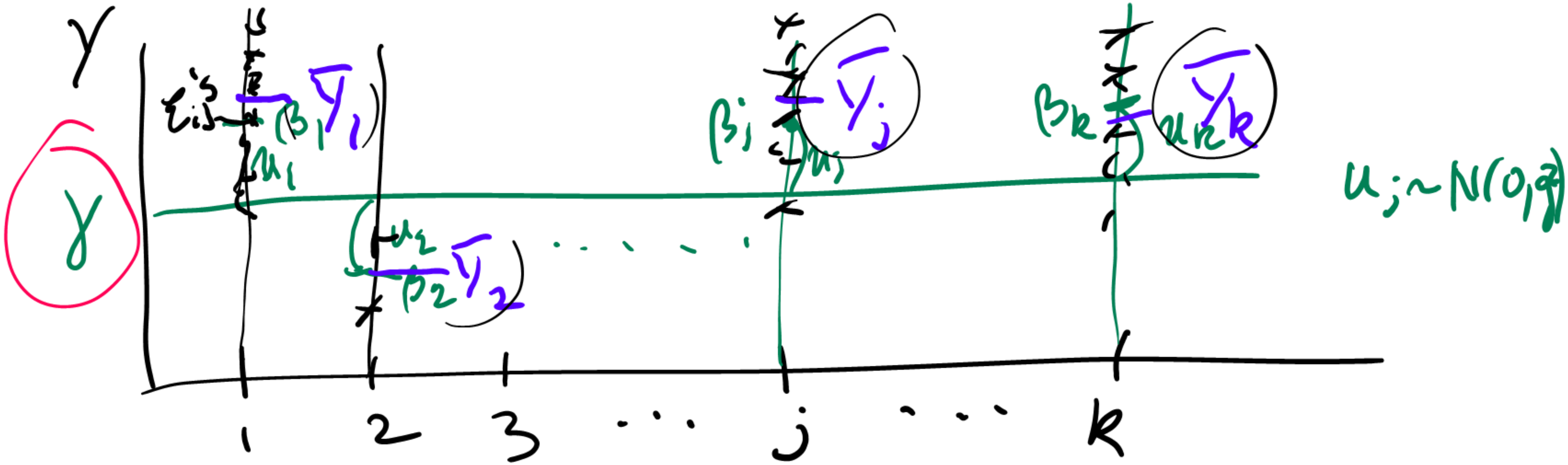
$Var(u_i) = g^2$

$Var(\varepsilon_{ij}) = \sigma^2$

lme(Y ~ 1, dd

random = ~ 1 | school

$$Y_{ij} = \beta_i + \varepsilon_{ij} \qquad \beta_i = \gamma + u_i$$



Estimate γ:

1) $\bar{Y}_{GM} = \dfrac{\sum Y_{ij}}{\sum n_i} = \dfrac{\sum_i n_i \bar{Y}_i}{\sum n_i} = \sum_{i=1}^{k} w_i \bar{Y}_i$

$w_i \propto n_i$

2) $\bar{Y}_{\cdot\cdot} = \dfrac{\sum \bar{Y}_i}{k} = \sum_{i=1}^{k} \frac{1}{k} \bar{Y}_i$

3) $\bar{Y}_1$ est of γ $\quad \bar{Y}_2$ est of γ $\ldots \bar{Y}_k$ est of γ

$\bar{Y}_i$ unb for γ

$$\bar{Y}_i = \gamma + \underline{u_i} + \underline{\frac{\sum_j \varepsilon_{ij}}{n_i}} \qquad \text{EVER}$$

$$E(\bar{Y}_i) = \gamma + 0 + 0 = \gamma$$

$\bar{Y}_i$'s are independent.

Need weights proportional to inverse of Variance

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Need weights proportional to inverse of Variance

... 100 Times

$$\bar{Y}_i = \gamma + \alpha_i + \frac{\sum \varepsilon_{ij}}{n_i}$$

$$\mathrm{Var}(\bar{Y}_i) = g^2 + \frac{\sigma^2}{n_i}$$

$$w_i \propto \frac{1}{g^2 + \frac{\sigma^2}{n_i}} \propto \frac{1}{\frac{g^2}{\sigma^2} + \frac{1}{n_i}}$$

If n_i equal

$w_i \propto$ constant

i.e are all equal

1) $\frac{g^2}{\sigma^2}$ very small

$$w_i \propto \left(\cancel{\frac{g^2}{\sigma^2}}^{\,0} + \frac{1}{n_i}\right)^{-1}$$

$$\propto \left(\frac{1}{n_i}\right)^{-1} = n_i$$

i.e $\overline{Y}_{GM}$

2) $\frac{g^2}{\sigma^2}$ very large

$$w_i \propto \left(\frac{g^2}{\sigma^2} + \frac{1}{n_i}\right)^{-1} = \text{constant}$$

i.e. $\overline{Y}_{..} = \frac{\sum \overline{Y}_i}{k}$

3) Estimate $\frac{g^2}{\sigma^2}$

Then use $w_i \propto \left(\frac{g^2}{\sigma^2} + \frac{1}{n_i} \right)^{-1}$

Mixed Model

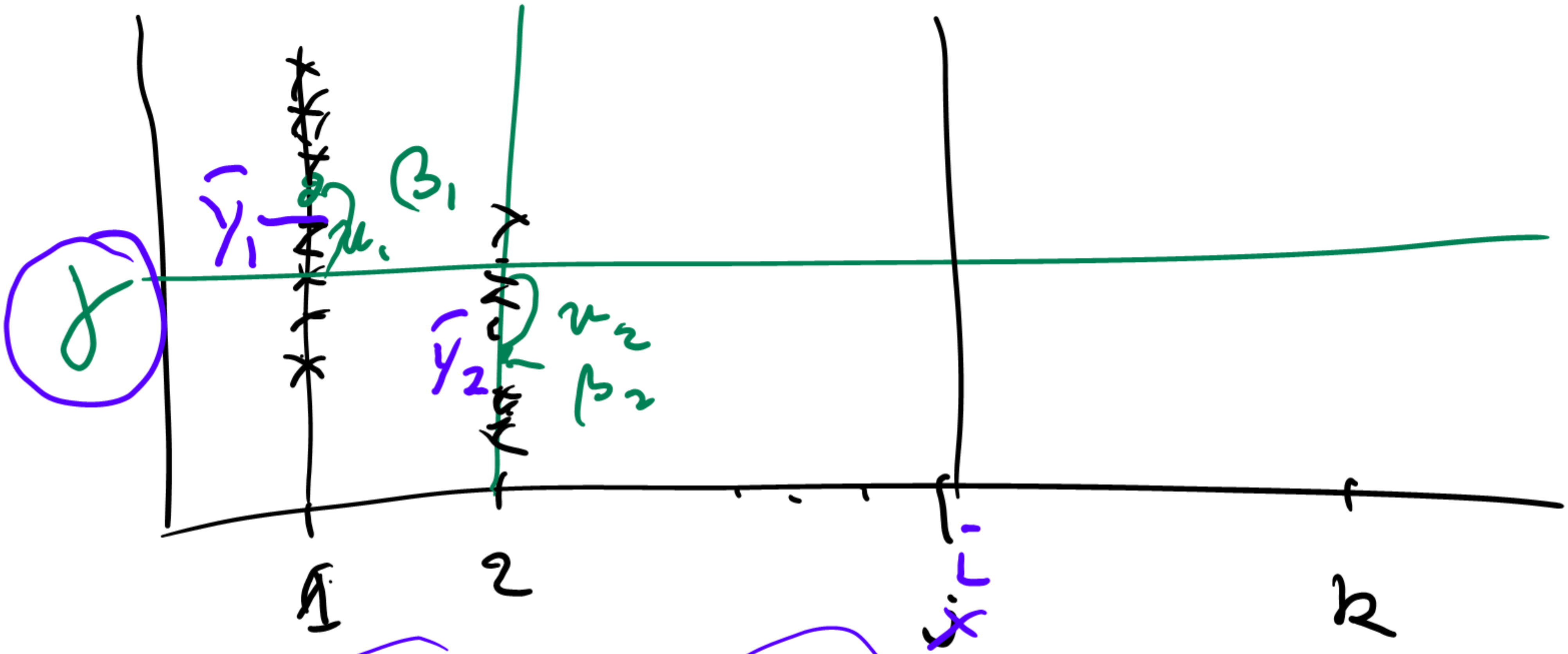
$$\hat{\gamma}_P = \frac{\sum Y_{ij}}{\sum n_{ij}} = \bar{Y}_{G \cdot \cdot}$$

$$\hat{\gamma}_{FE} = \frac{\sum \bar{Y}_i}{k}$$

$$\gamma_{\text{naive}} = \sum w_i \bar{Y}_i$$

γ_P γ_{naive} γ_{FE} possible

$$E(\gamma_{\text{naive}})$$



$\bar{Y}_i$ est of β_i

cond'l

$E(\bar{Y}_i \mid \beta_i) = \beta_i$ unbiased for β_i

$\bar{Y}_i$ is BLUE of $\beta_i \mid \beta_i$

$$\mathrm{Var}(\bar{Y}_i - \beta_i) = \mathrm{Var}\left(\cancel{\gamma} + \cancel{u_j} + \frac{\sum \varepsilon_{ij}}{n_i} - (\cancel{\gamma} + \cancel{u_j})\right)$$

$$= \mathrm{Var}\left(\frac{\sum \varepsilon_{ij}}{n_i}\right) = \frac{\sigma^2}{n_i}$$

<u>Mult reg</u> $\sigma^2 (X_i' X_i)^{-1}$

$$= \frac{\sigma^2}{n_i} \left(\Sigma_{x_i}\right)^{-1}$$

Guess β_i - no info except γ

$E(\beta_i - \gamma) = 0$

γ "est" of β_i

$$\beta_i = \gamma + \eta_i$$

$$\mathrm{Var}(\beta_i - \gamma) = g^2$$

Two guesses $\bar{Y}_i$ σ^2/n_i

γ g^2

Prediction

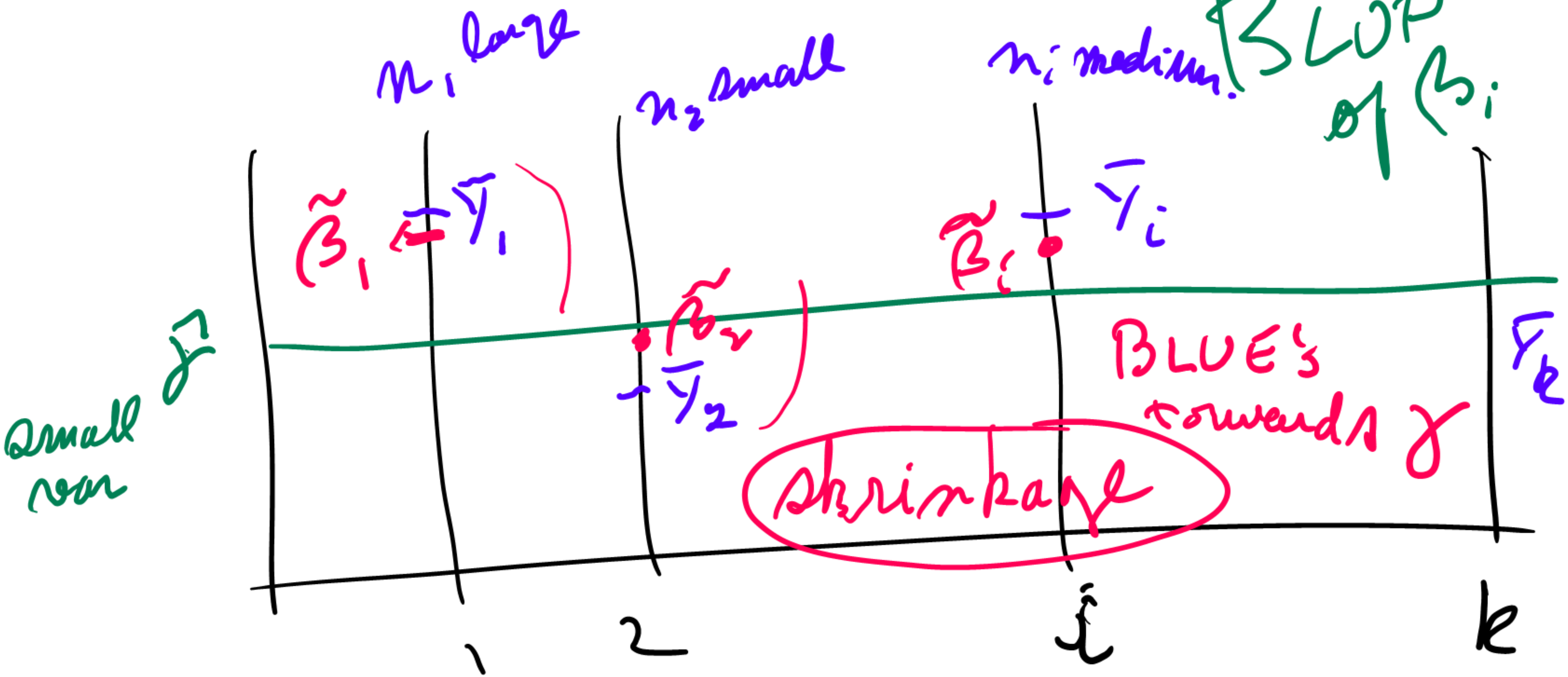
β_i from Y_{ij}'s & γ

BLU~~E~~ Predictor of β_i

random

$$\left(\frac{1}{g^2}+\frac{1}{\sigma^2/n_i}\right)^{-1}\left(\frac{1}{g^2}\gamma+\frac{n_i}{\sigma^2}\bar{Y}_i\right)=\tilde{\beta}_i$$

BLUP of β_i



fit <- lme(. . . .)

β's : coef(fit)

u's ranef(fit)

$\hat{\gamma}$ fixef(fit)

$\hat{\eta}_1, \hat{\eta}_2, \dots \hat{\eta}_p \qquad E(\eta_i) = \eta$

$$\mathrm{Var}\begin{pmatrix}\hat{\eta}_1\\ \hat{\eta}_2\\ \vdots\\ \hat{\eta}_p\end{pmatrix} = V$$

$$(1'V^{-1}1)^{-1}1'V^{-1}\hat{y} \longrightarrow \text{Check}$$

$$\mathrm{Var}(Y) = E(\mathrm{Var}(Y|X)) + \mathrm{Var}(E(Y|X))$$