$$y = X_1\hat{\beta}_1 + X_2\hat{\beta}_2 + e \qquad e \perp \mathcal{L}(X_1, X_2)$$

$$Q_2 = I - P_2 \qquad \underline{P_2} = \underset{n\times p_2}{X_2}\,\underset{p_2\times p_2}{(X_2'X_2)^{-1}}\,\underset{p_2\times n}{X_2'}$$

P_2

$$rk(P_2) = p_2$$

1) $P_2' = P_2$

2) $$P_2P_2 = X_2(X_2'X_2)^{-1}\cancel{X_2'X_2(X_2'X_2)^{-1}}\underline{X_2'}$$

$$= X_2(X_2'X_2)^{-1}X_2'$$

$$= P_2$$

$P_2^2 = P_2$ (P_2 idempotent)

$$y = X_1\hat{\beta}_1 + X_2\hat{\beta}_2 + e \qquad e \perp \mathcal{L}(X_1, X_2)$$

$$Q_2 = I - P_2 \qquad \underline{P_2} = \underset{n\times p_2}{X_2}\underset{p_2\times p_2}{(X_2'X_2)^{-1}}\underset{p_2\times n}{X_2'}$$

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$$= P_2$$

$P_2^2 = P_2$ (P_2 idempotent)

② show $X_2'(Y - P_2 Y) = 0$

$X_2'(Y - P_2 Y) = X_2' Y - X_2' P_2 Y$

But $P_2 X_2 = X_2 (X_2' X_2)^{-1} X_2' X_2$

$= X_2$

$= X_2' Y - X_2' Y = 0$ ✓

P_2 is $\boxed{\text{m o p}}$ onto $\mathcal{L}(X_2)$

Facts: 1) $\underset{\sim}{x} \in \mathcal{L}(X_2)$ $\quad P_2 \underset{\sim}{x} = x$

2) $\underset{\sim}{x} \in \mathcal{L}^{\perp}(X_2)$ i.e. $X_2' \underset{\sim}{x} = 0$

$P_2 x = X_2 (X_2' X_2)^{-1} X_2' x = \underset{\sim}{0}$

$tr(A) = tr(A')$

$tr(AB) = tr(BA)$

3) If P_2 is a mop then

$rank(P_2) = tr(P_2)$

$tr(X_2(X_2'X_2)^{-1}X_2')$

$= tr(X_2'X_2(X_2'X_2)^{-1})$

$= tr(I) = p_2$

4) $Q_2 = I - P_2$ is mop onto $\mathcal{L}^{\perp}(X_2)$

Regression $\quad \underset{\sim}{Y} = X\underset{\sim}{\beta} + \underset{\sim}{\varepsilon}$

$P = X(X'X)^{-1}X'$ mop$(\mathcal{L}(X))$ $\quad \underset{\sim}{\varepsilon} \sim N(\underset{\sim}{0}, \sigma^2 I)$

$Q = I - P$ " $(\mathcal{L}^{\perp}(X))$

$$\hat{y} = \underbrace{P}\, Y \qquad \underset{\sim}{e} = \underline{Q}\, Y$$

$$SSE = \hat{\underset{\sim}{e}}'\underset{\sim}{e} = Y'\underbrace{\boxed{Q'Q}}_{Q}Y = Y'QY$$

$$X^* = XA \qquad A^{-1} \text{ exists}$$

if $\mathcal{L}(X^*) = \mathcal{L}(X)$ and both are basis.

$$X^*(X^{*\prime}X^*)^{-1}X^{*\prime} = X\boxed{\underline{A}\,\underline{(A'X'XA)}^{-1}A'}X'$$

$$= X\left[A(A'X'XA)^{-1}A'\right]X'$$

$$= X(AA^{-1})(X'X)^{-1}(A'^{-1}A')X'$$

$$= X(X'X)^{-}X' = P$$

AVP

$$Y = X_1 \hat{\beta}_1 + X_2 \hat{\beta}_2 + e$$

$$\underline{Q_2 Y} = \underline{Q_2 X_1 \hat{\beta}_1} + \underline{Q_2 X_2 \hat{\beta}_2} + Q_2 e$$

$$Y_{\cdot 2} = X_{1\cdot 2} \hat{\beta}_1 + 0 \hat{\beta}_2 +$$

$e \in \mathcal{L}^\perp(X_1, X_2)$

Q_2 map onto $\mathcal{L}^\perp(X_2) \supset \mathcal{L}^\perp(X_1, X_2)$

$e \in \mathcal{L}^\perp(X_1, X_2) \subset \mathcal{L}^\perp(X_2)$

$\therefore Q_2 e = e$

$\therefore$ $\underline{Q_2 Y} = \underline{Q_2 X_1} \underline{\hat{\beta}_1} + e$

WTS $e \perp \mathcal{L}(Q_2 X_1)$

$e' Q_2 X_1$

$= e' X_1 = 0$

$SSE = e'e$

$\underline{e} \in \underline{\mathcal{L}^{\perp}(X_1, X_2)}$

Q_2 proj onto $\mathcal{L}^{\perp}(X_2)$

$Y \sim X_1 + X_2$

$Y_{\cdot 2} \sim X_{1\cdot 2}$

$Y \sim X_1$

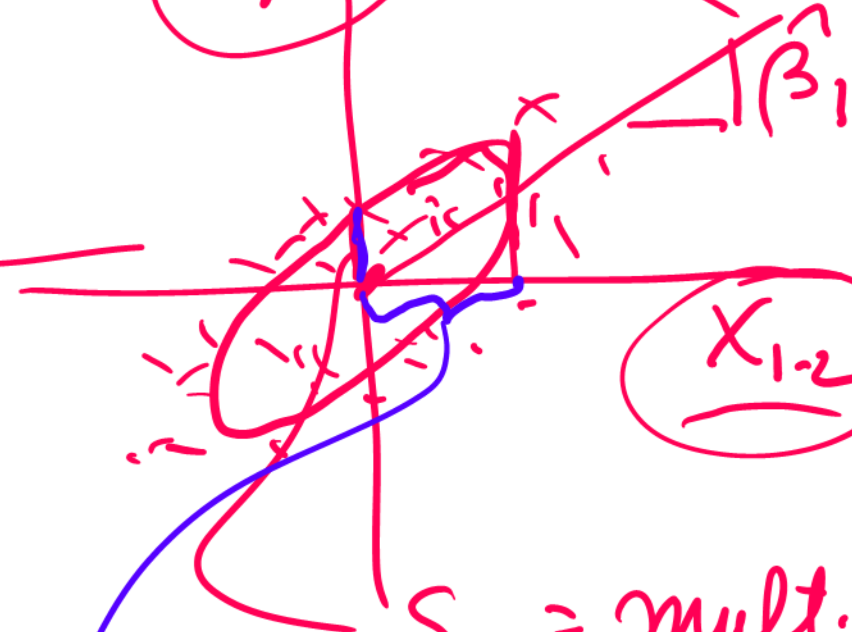
$Y \sim X_1 + \hat{X}_{1\cdot 2}$

pred of X_1 using X_2

AV/P

Propensity score

causal inference



S_e = mult. reg. S_e

$S_{X_1|X_2}$

$$\widehat{SD}(\hat\beta_1) = \frac{1}{\sqrt{n}} \frac{S_e}{S_{X_1|X_2}}$$

HOLDS for M Reg

$$VIF = \frac{S^2_{x_1}}{S^2_{x_1|x_2}}$$ ← Simple ← multiple.

$$SDIF = \sqrt{VIF} \approx \sqrt{\frac{Var(\hat{\beta}_1)}{Var(\hat{y}_1)}}$$ ← MR ← Simple reg

---

$$Y \sim X_1 + \hat{X}_{1.2}$$

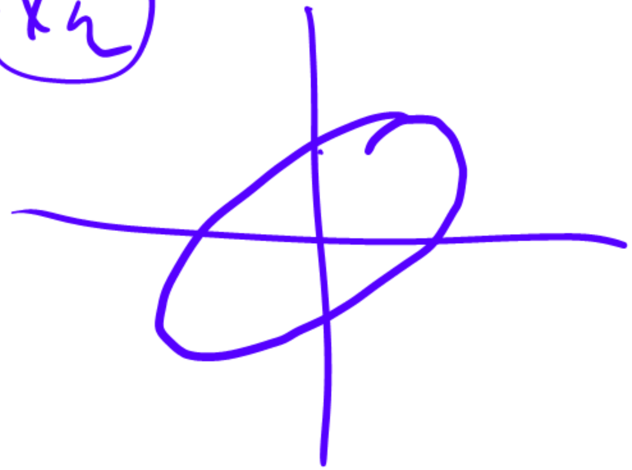
AVP resid of Y on $\hat{X}_{1.2}$ (instead of X_2)

$\Longleftrightarrow$ resid of X_1 on $\hat{X}_{1.2}$ = resid of X_1 on X_2

$$X_{1.2} = X_1 - \hat{X}_{1.2}$$

$Y \sim X_1 + X_2$

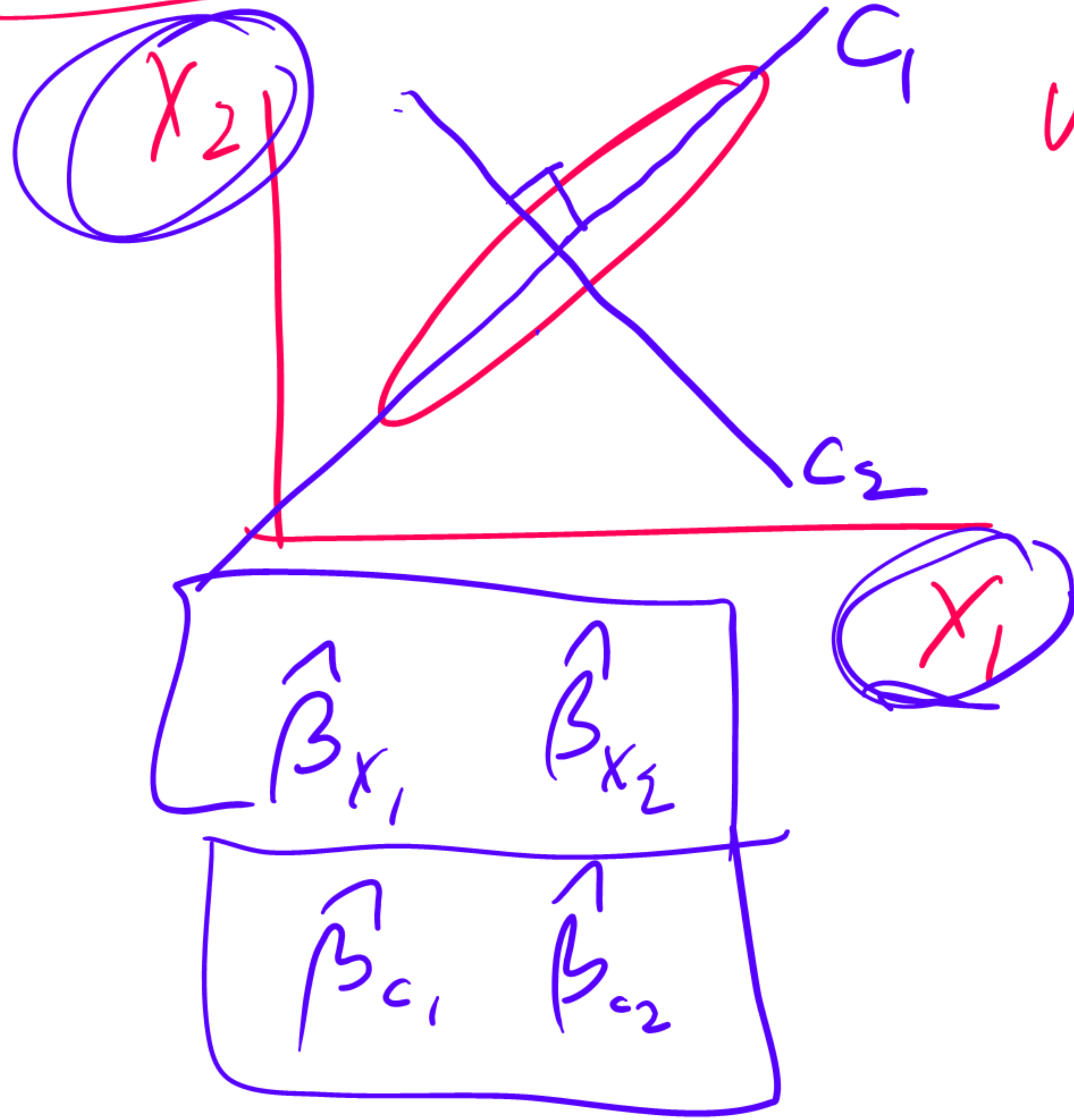
$Y \sim X_1 + \hat{X}_{1.2}$



same

Should we choose X_2 to make $SD(\hat{\beta}_1)$ small

USE PC



$Y \sim C_1 + C_2$

VIF = 1

$Y \sim X_1 + \hat{X}_{1\cdot 2}$ Same $\hat{\beta}_1$ as $Y \sim X_1 + X_2$

s_e larger than s_e

$s_{x_1|x_2}$ same as

$Y \sim X_1 + \hat{X}_{1\cdot 2} + \text{part } X_2$

keep $\hat{\beta}_1$ but $s_e \downarrow$