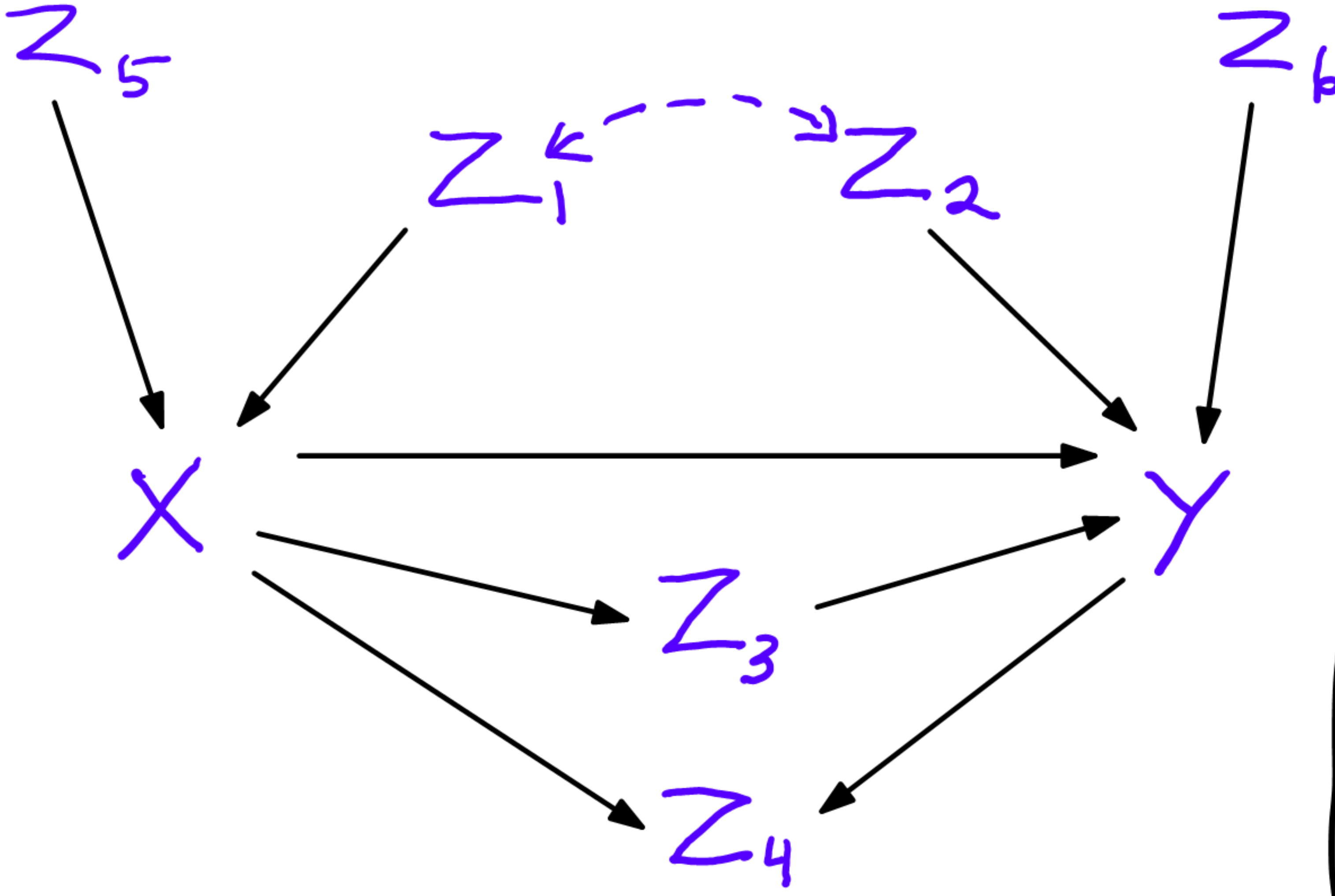
Does X cause Y?

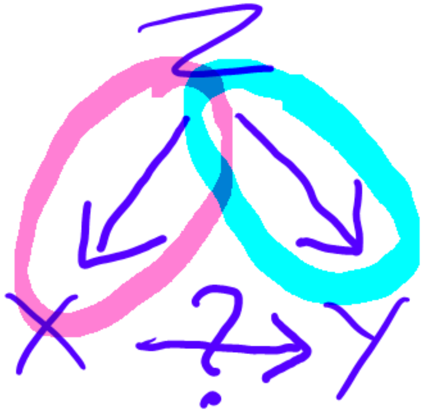
- A bird's eye view of methods with observational data
- Lord's Paradox and the role of longitudinal data

Causal Graph

Pearl & Mackenzie (2019)

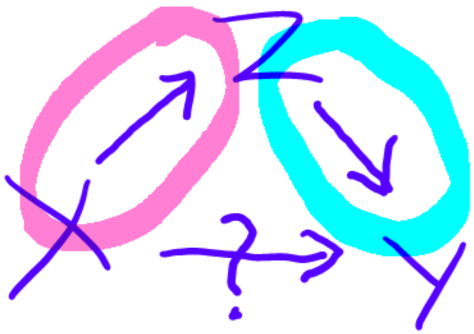


DAG = Directed acyclic graph



confounder

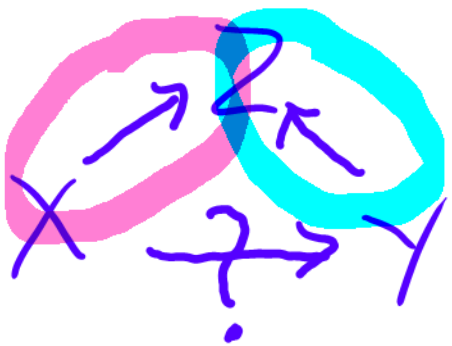
must **include** to see
see causal effect
of X on Y



mediator

must **exclude**

including may
wipe out a true effect



collider

e.g.
selection

must **exclude**

including may
create the impression
of an effect although there is
none

Moderators?

Can have - Confounder-moderators
- mediator-moderators
- collider-moderators

Also mediator-colliders, etc
BUT not represented by DAGs

- So convenient visualizations of DAG are useful abstraction but limited in practice

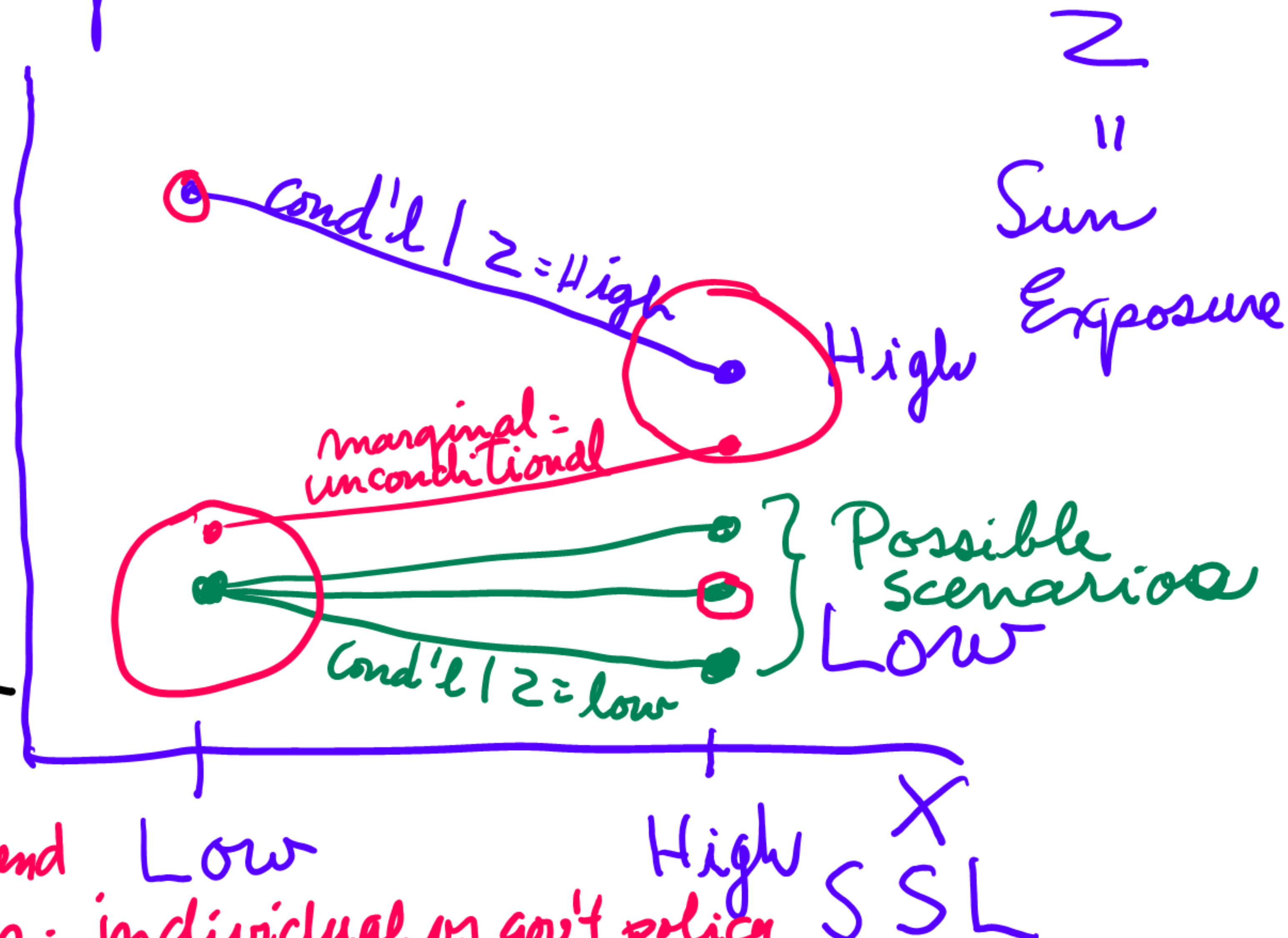
- avoiding inclusion of mediators more critical than avoiding colliders since inclusion of other confounders can correct for inclusion of a collider.

Moderator (= interaction) in data space

SSL example

Y

P(Skin Cancer)



"Moderator"

Cond'l lines not parallel

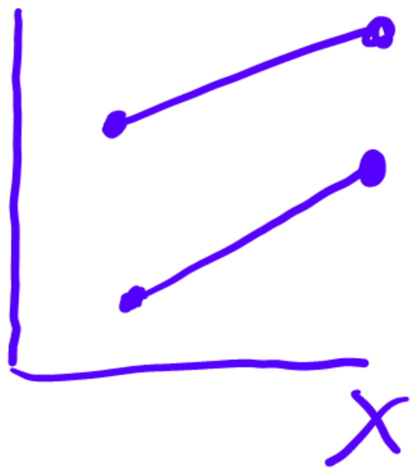
Role {mediator / confounder} of variable can depend on level of analysis: individual vs gov't policy

Moderation may be removable by monotonically transforming Y

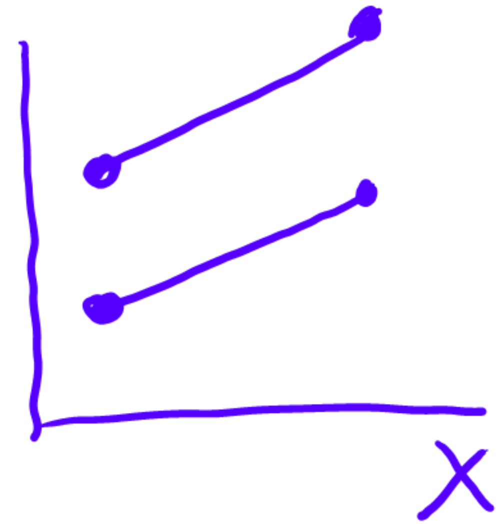
if 1) Cond'l effects in same direction

2) No crossings of lines

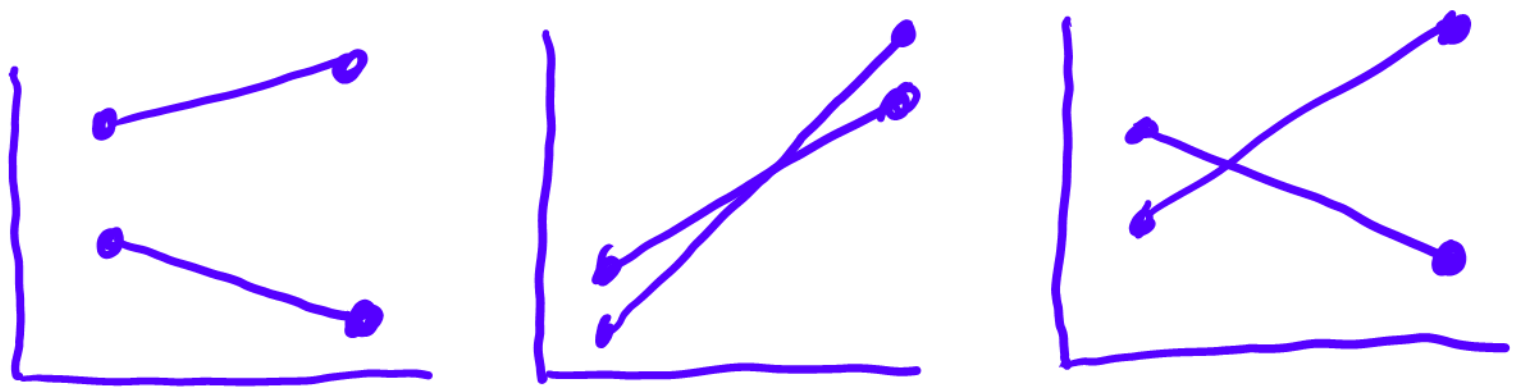
<u>Removable</u> $P(Y)$



log odds $= \log\left(\frac{P(Y)}{1-P(Y)}\right)$



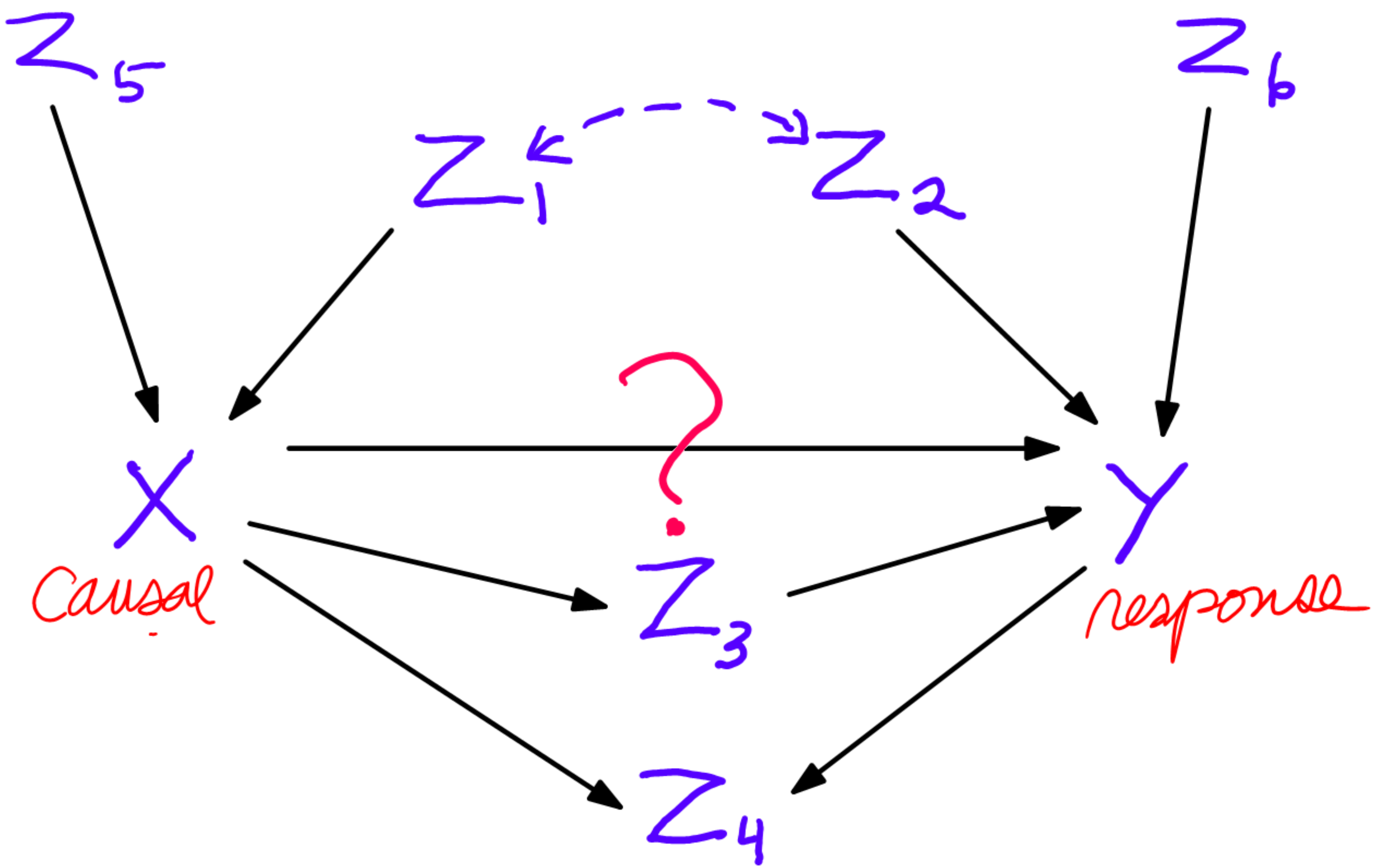
<u>Nonremovable</u>



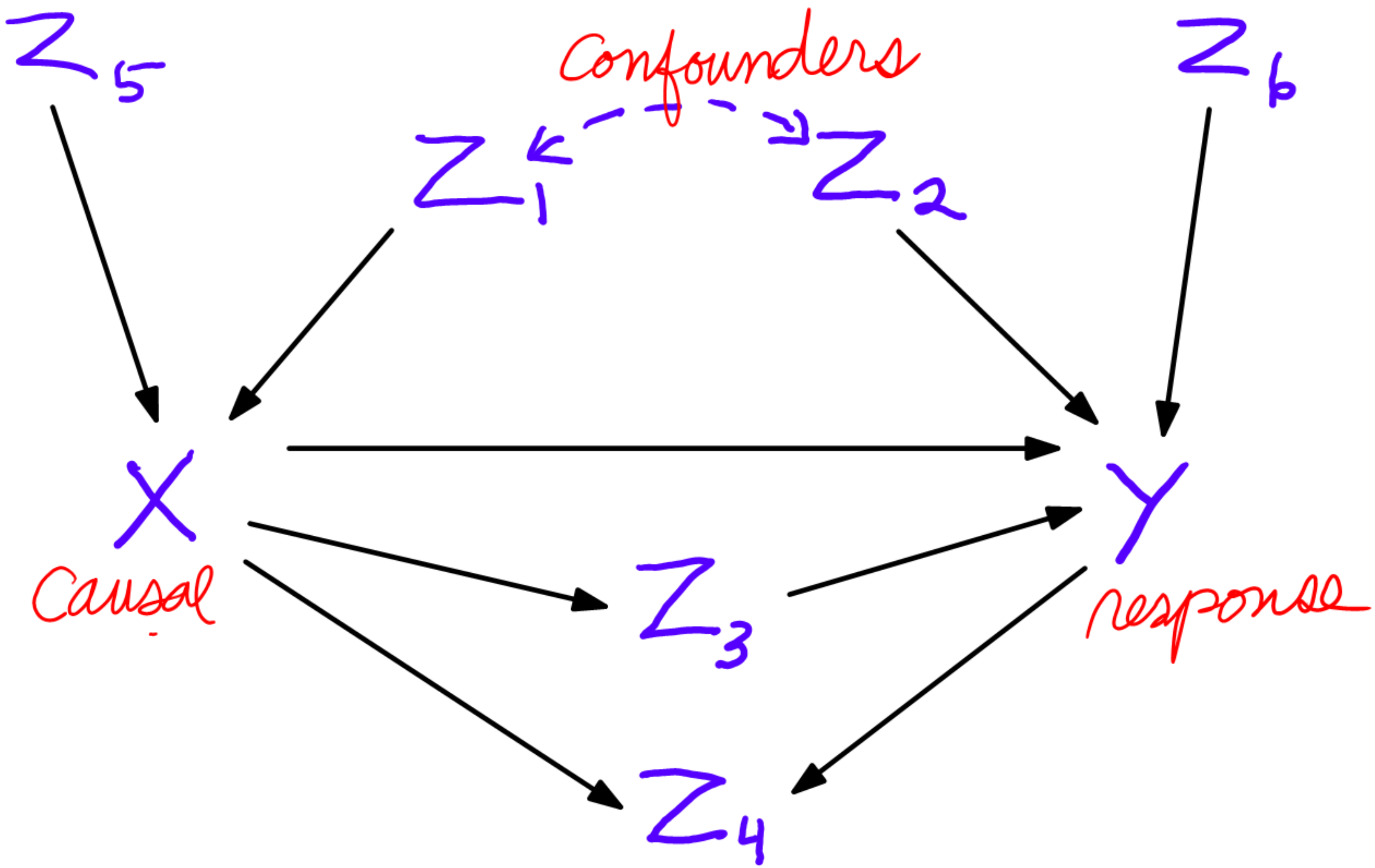
Note:

- Simpson effect: reversal of cond'l vs. marginal effects
- Moderation (= interaction in relation of X and Z with Y)
- Association (predictive)
- Causality

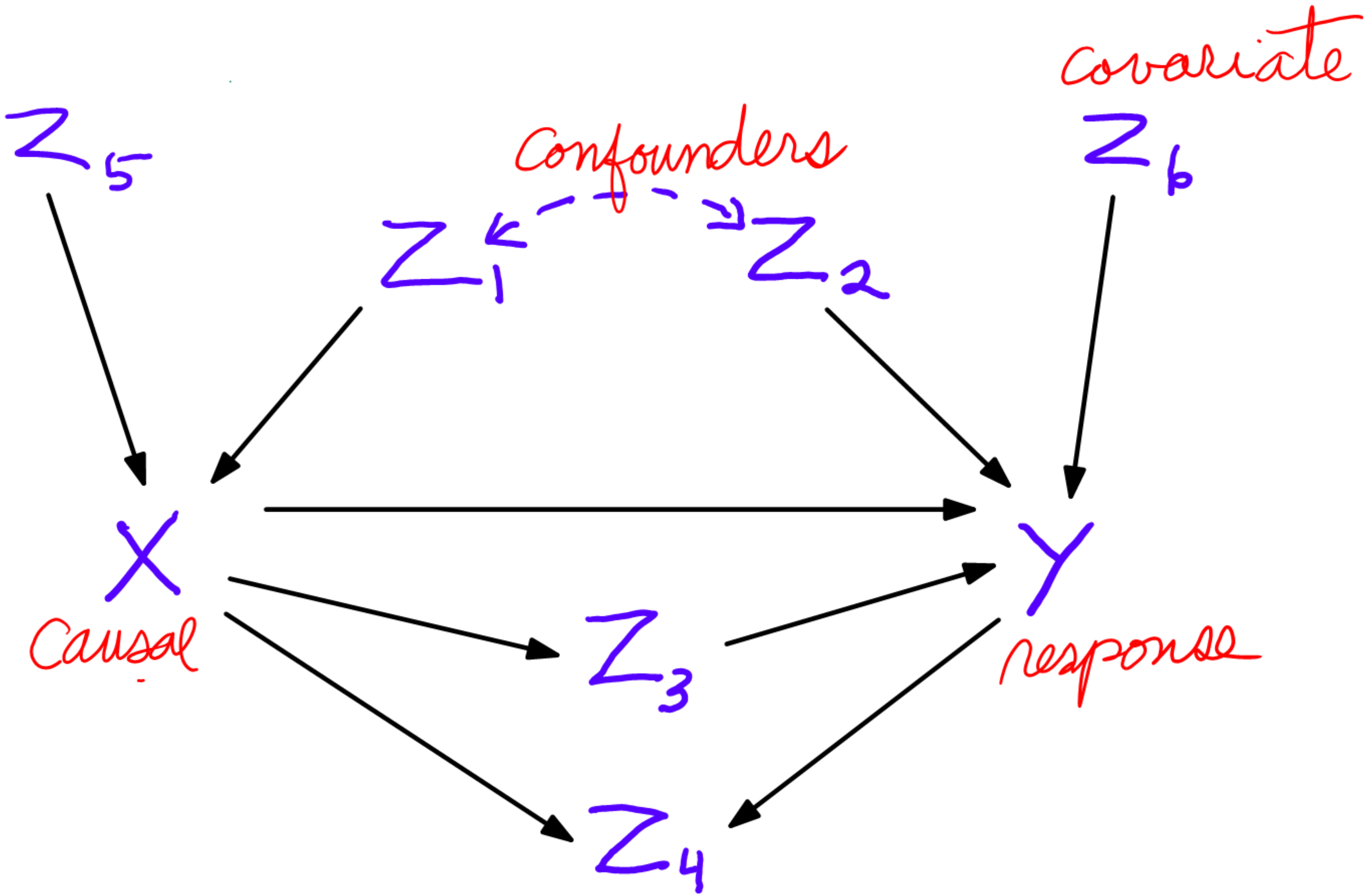
are distinct but not unrelated concepts



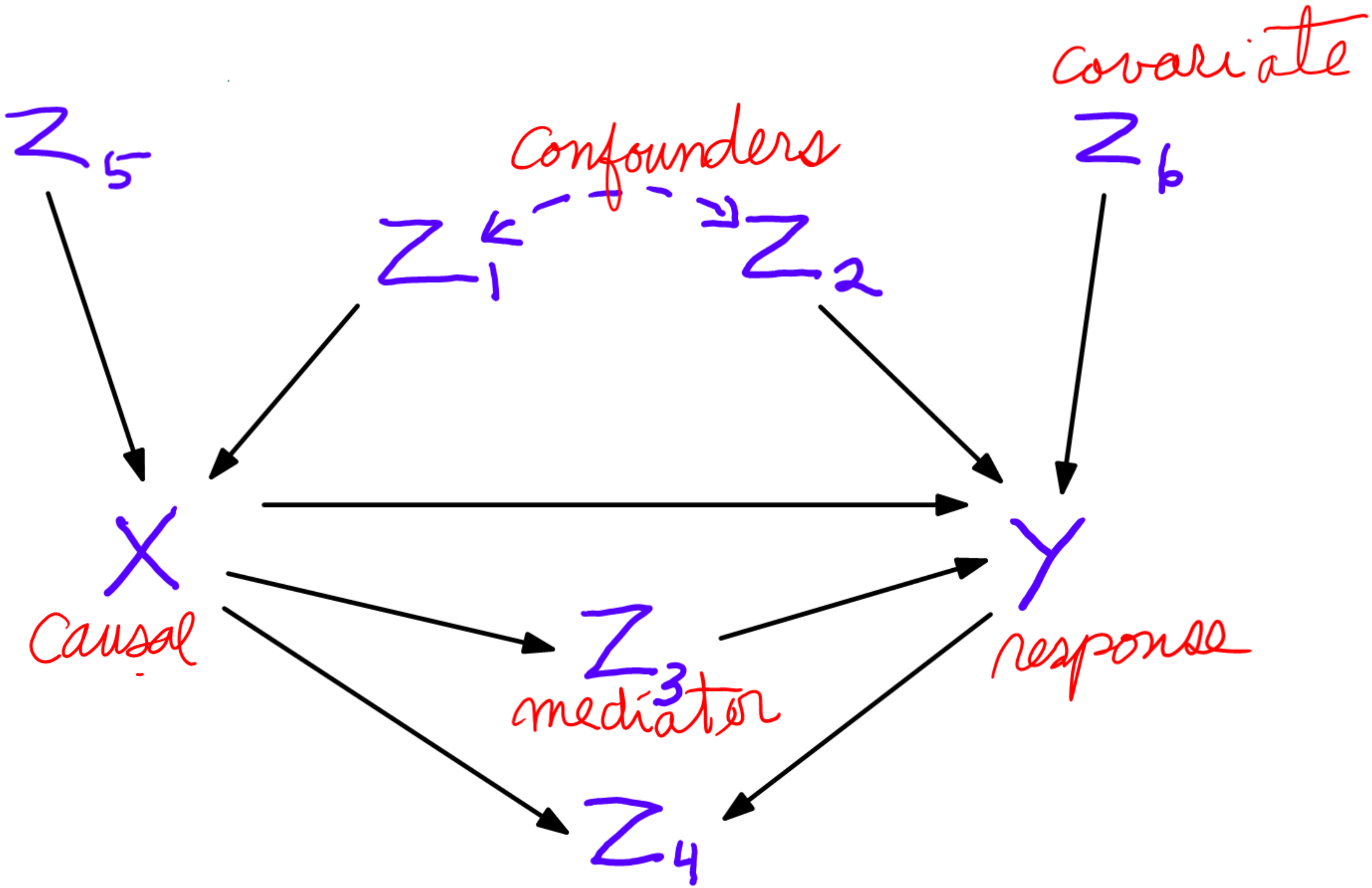
Z_5
Z_1
Z_2
Z_6
?
X
Y
Causal
response
Z_3
Z_4



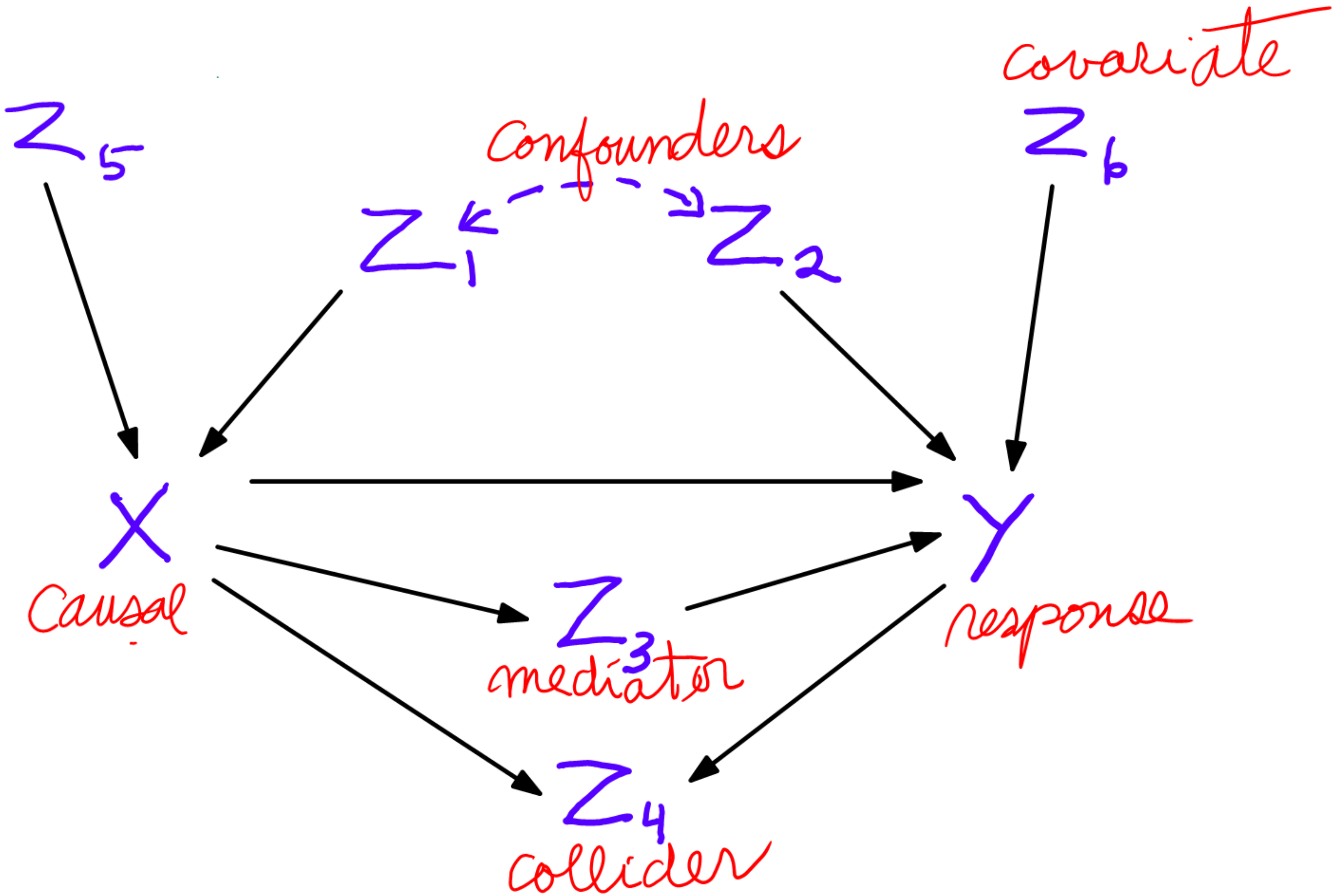
Confounders
Z_5
Z_1
Z_2
Z_6
X
Y
Causal
response
Z_3
Z_4

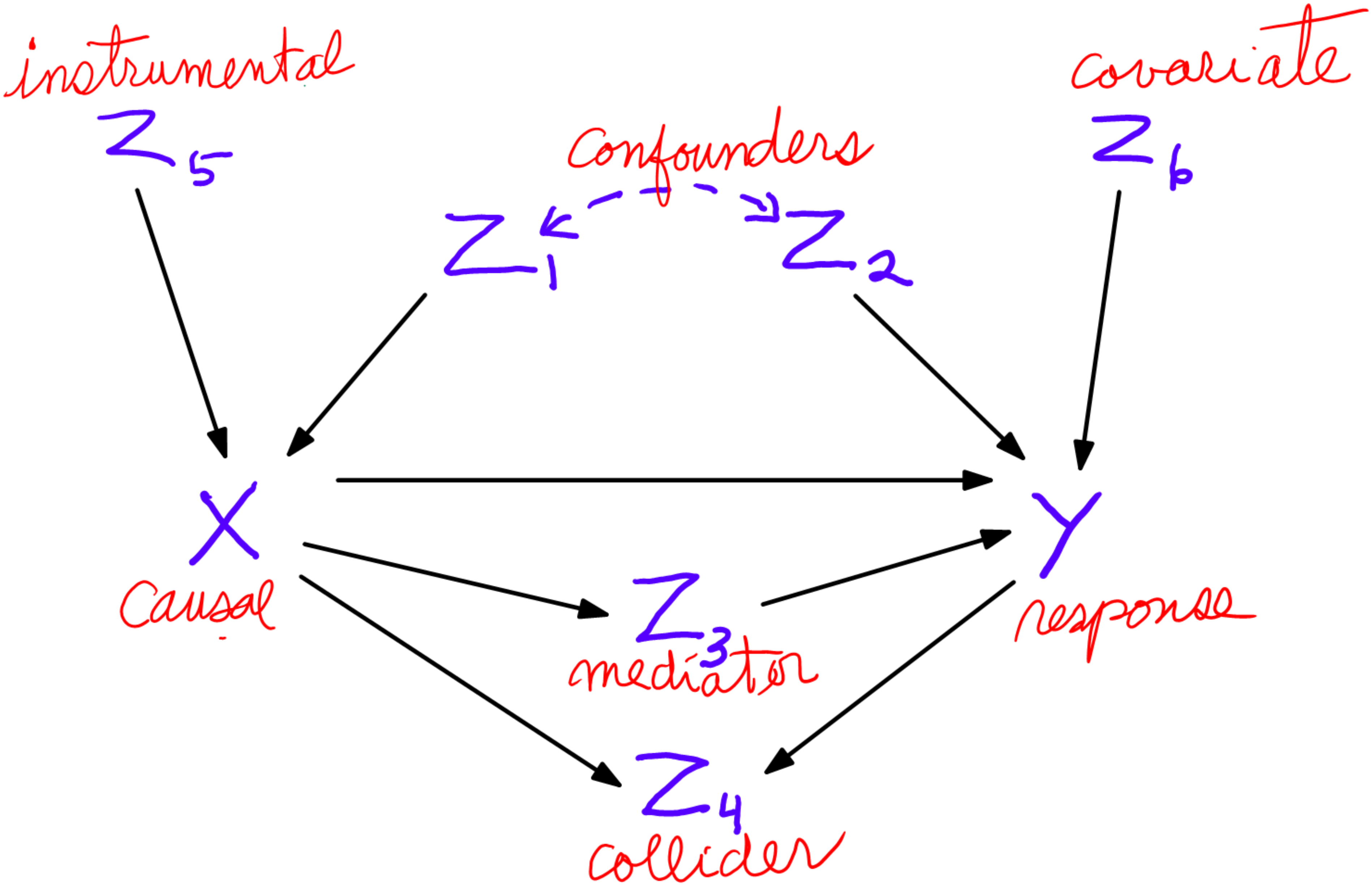


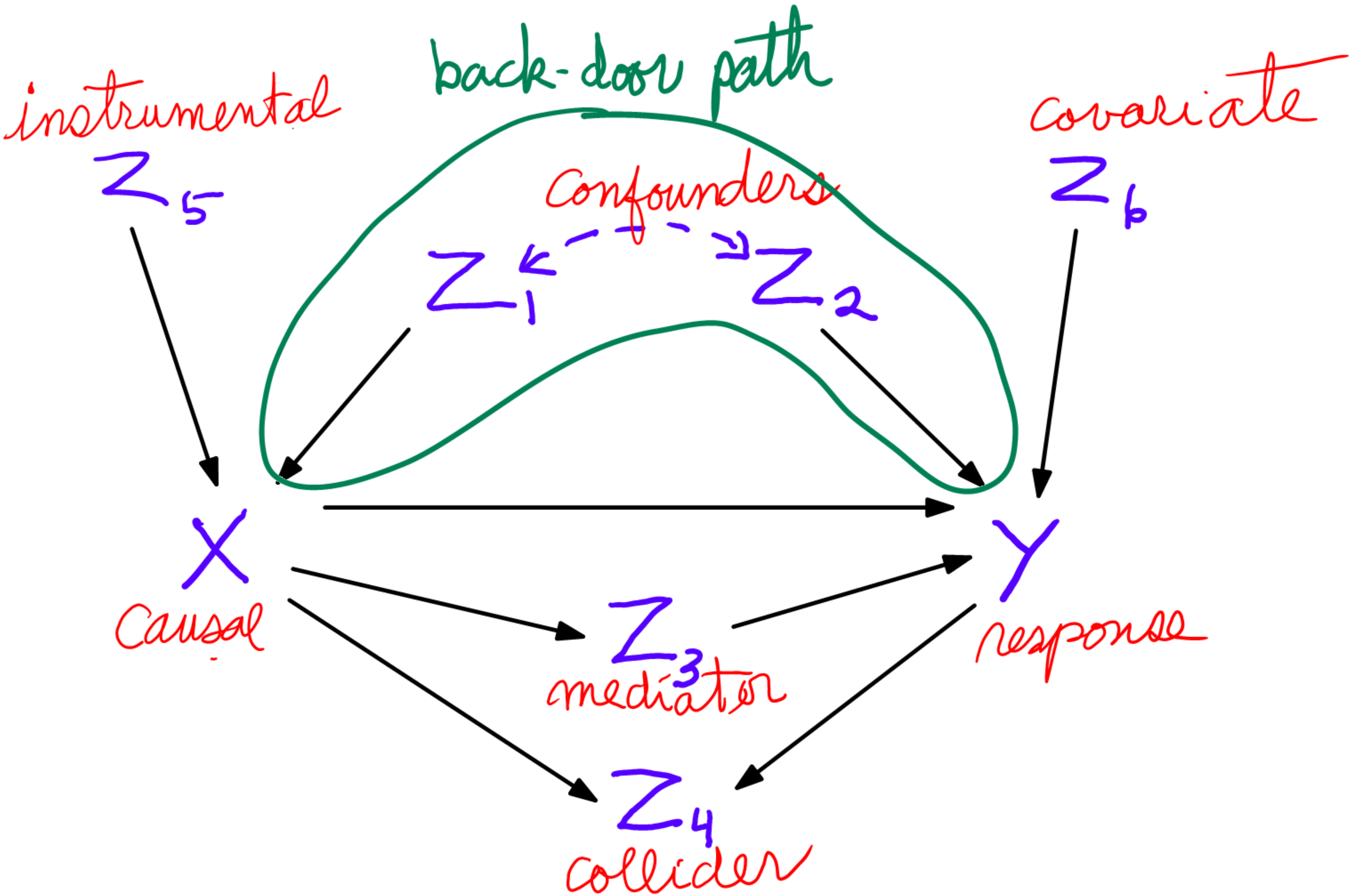
covariate
Z_5
Confounders
Z_1
Z_2
Z_6
X
Y
Causal
response
Z_3
Z_4



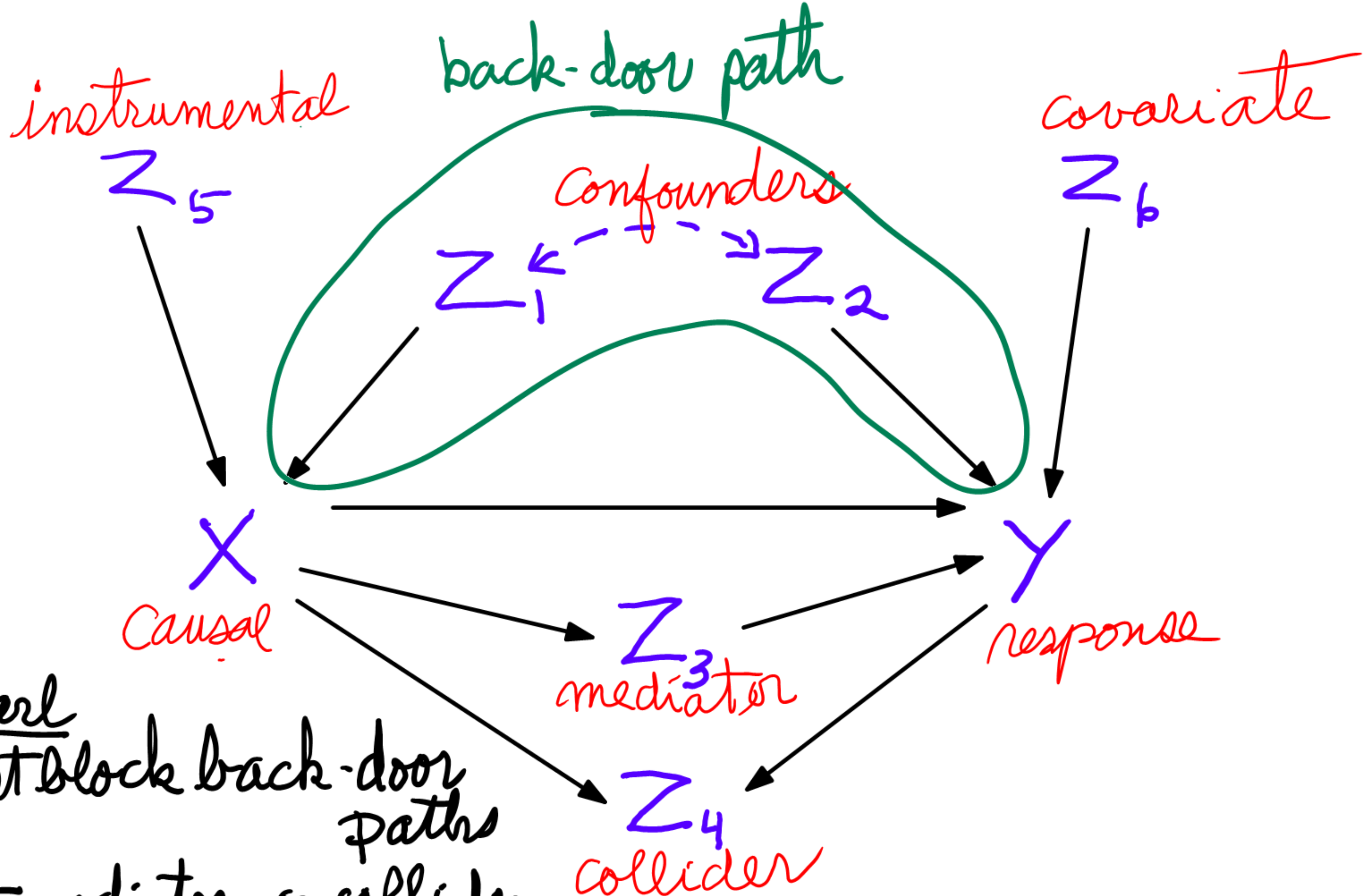
covariate
confounders
Z_5
Z_1
Z_2
Z_6
X
causal
Y
response
Z_3
mediator
Z_4


covariate
Z5
confounders
Z1
Z2
Z6
X
Y
causal
response
Z3
mediator
Z4
collider


instrumental
Z5
confounders
Z1
Z2
covariate
Z6
X
causal
Y
response
Z3
mediator
Z4
collider

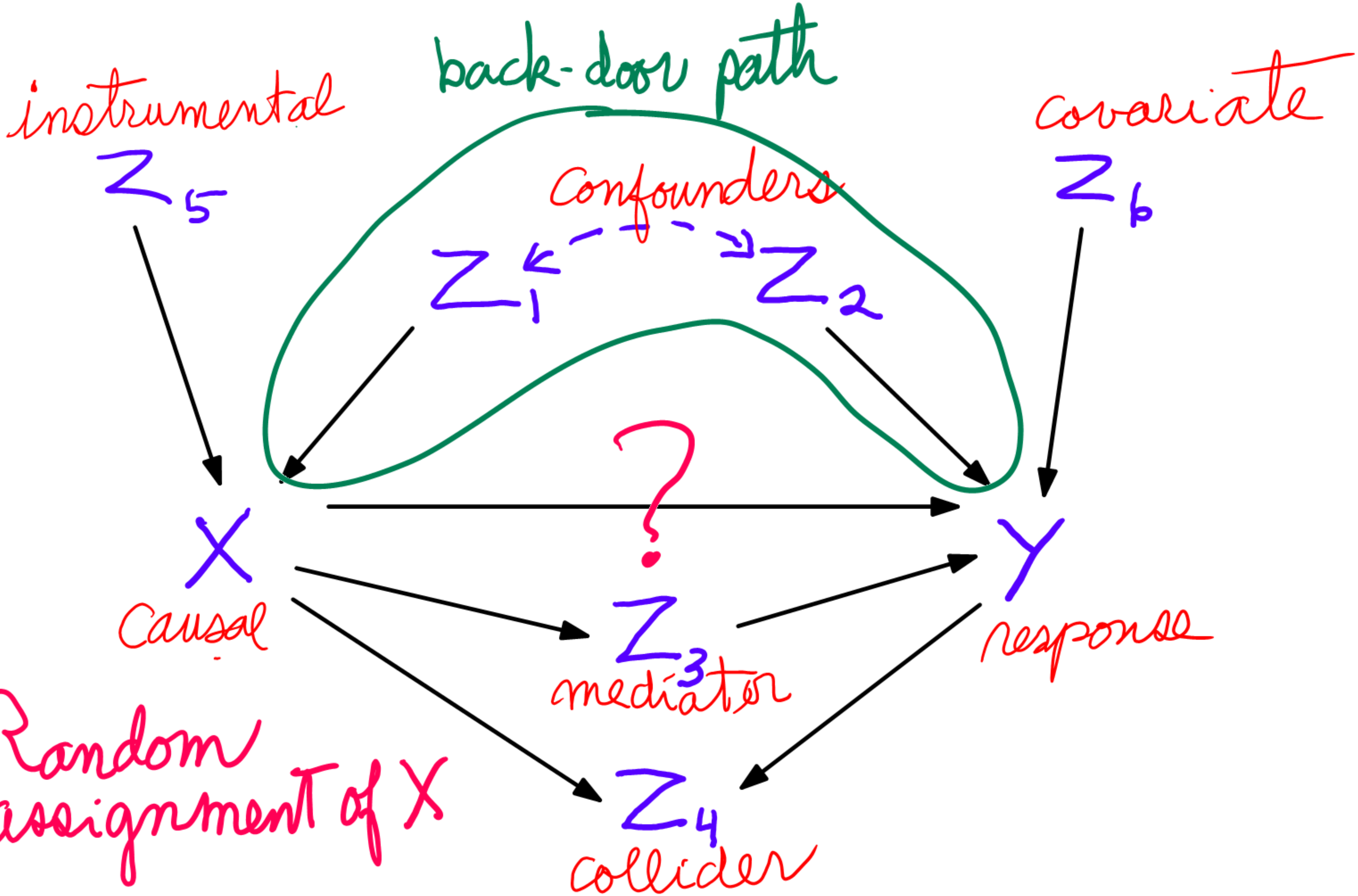


back-door path
instrumental
Z_5
confounders
Z_1
Z_2
covariate
Z_6
X
causal
Y
response
Z_3
mediator
Z_4
collider

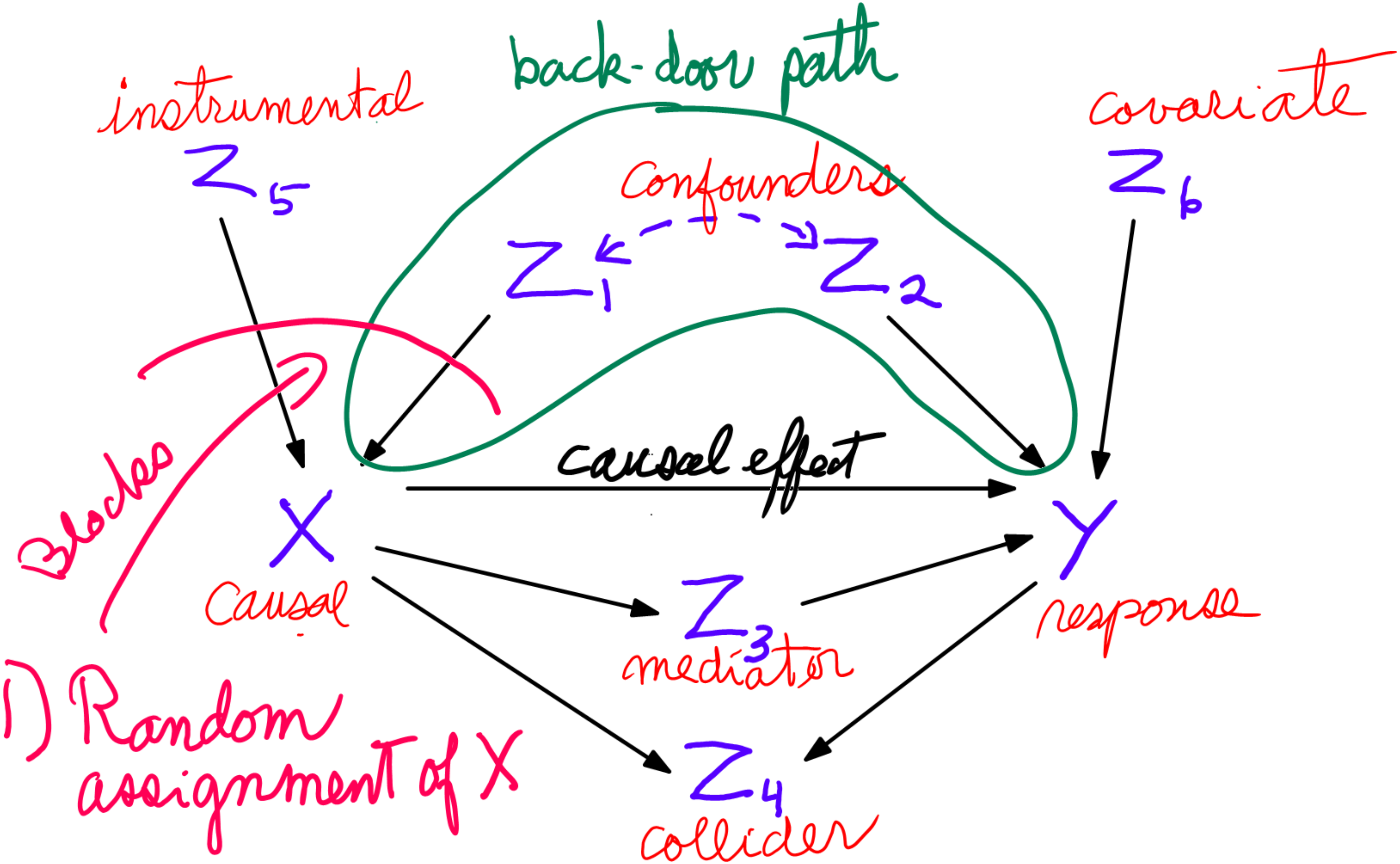


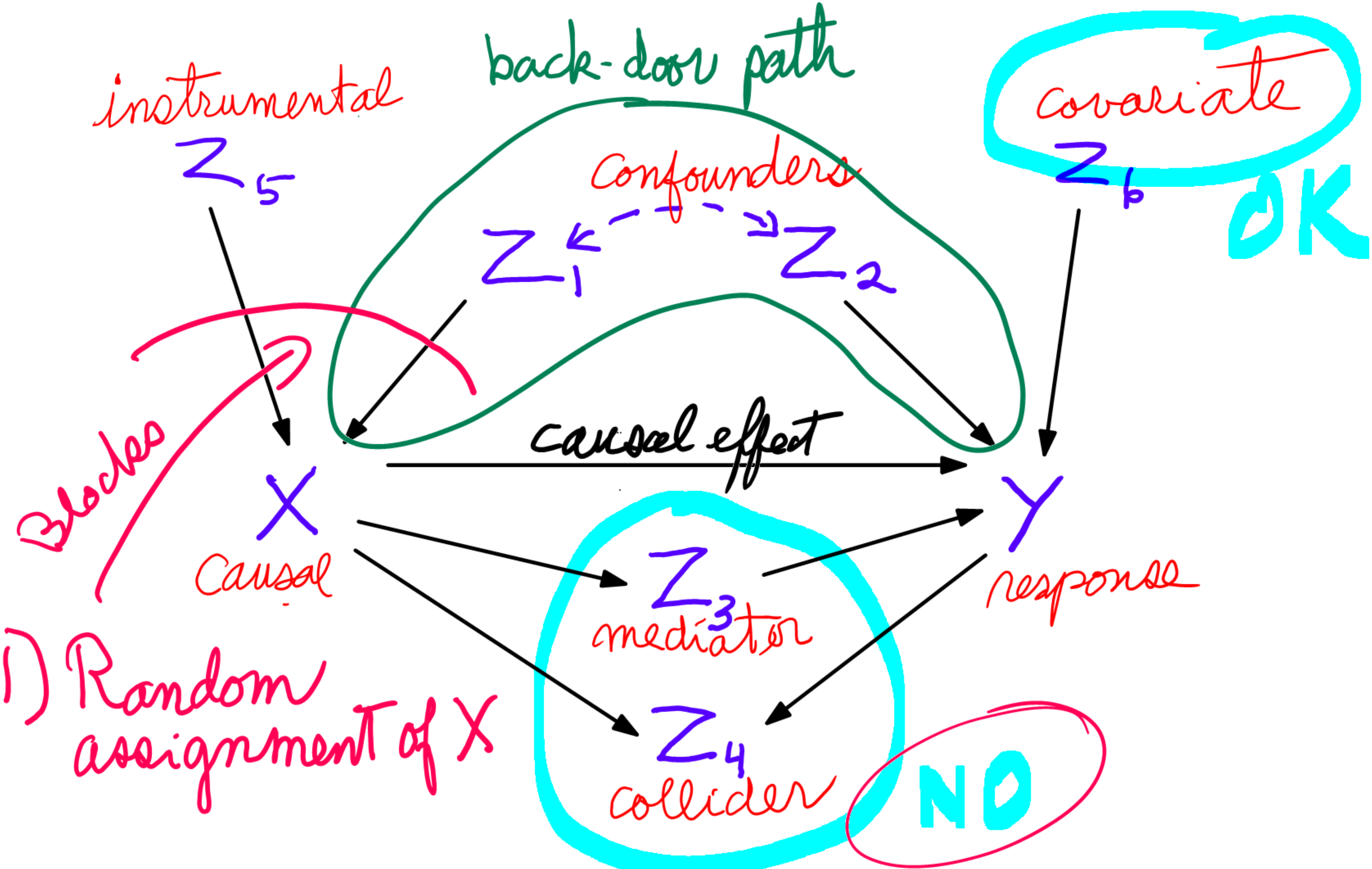
<u>Pearl</u>

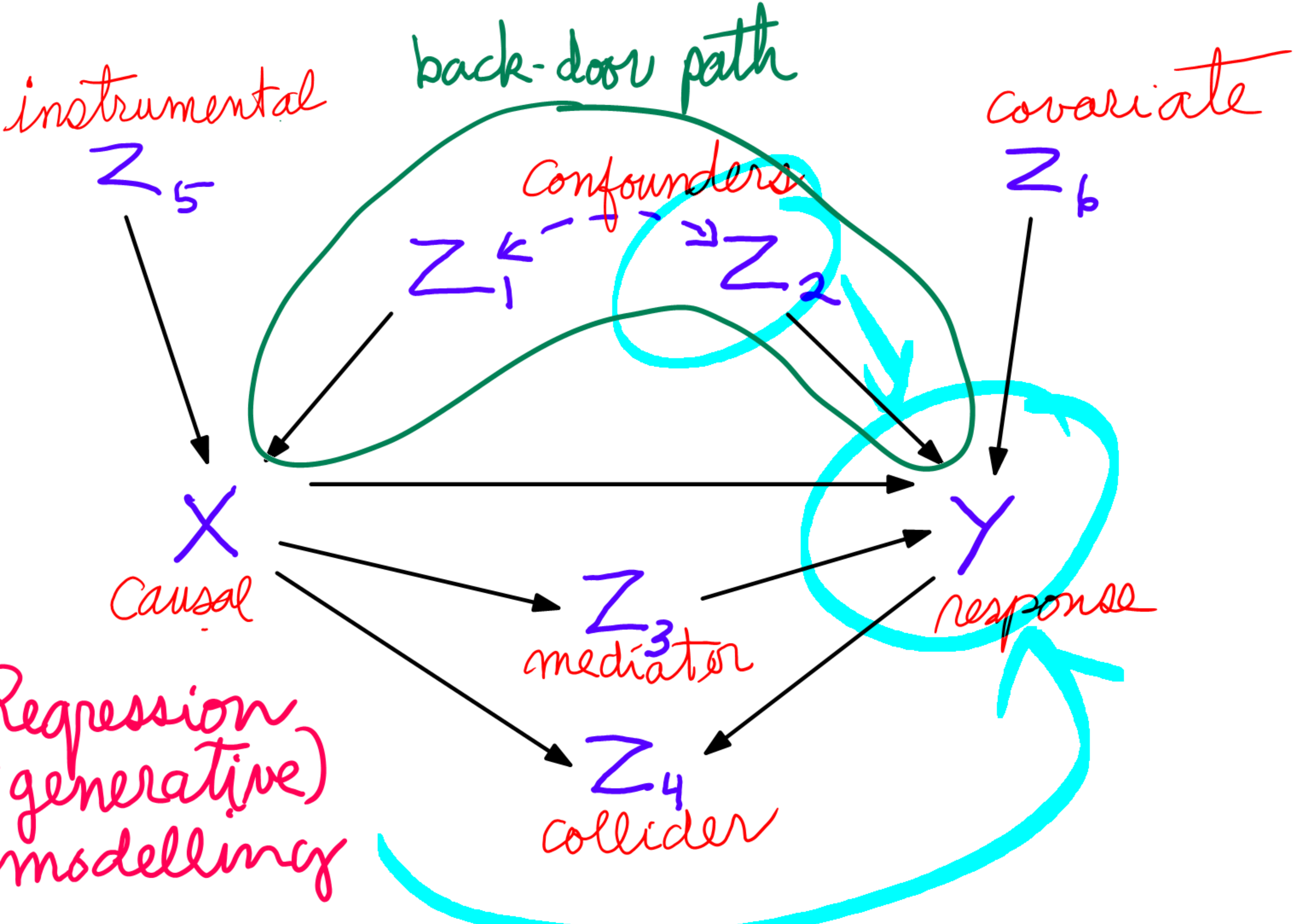
- Must block back-door paths
- <u>NOT</u> mediator or collider



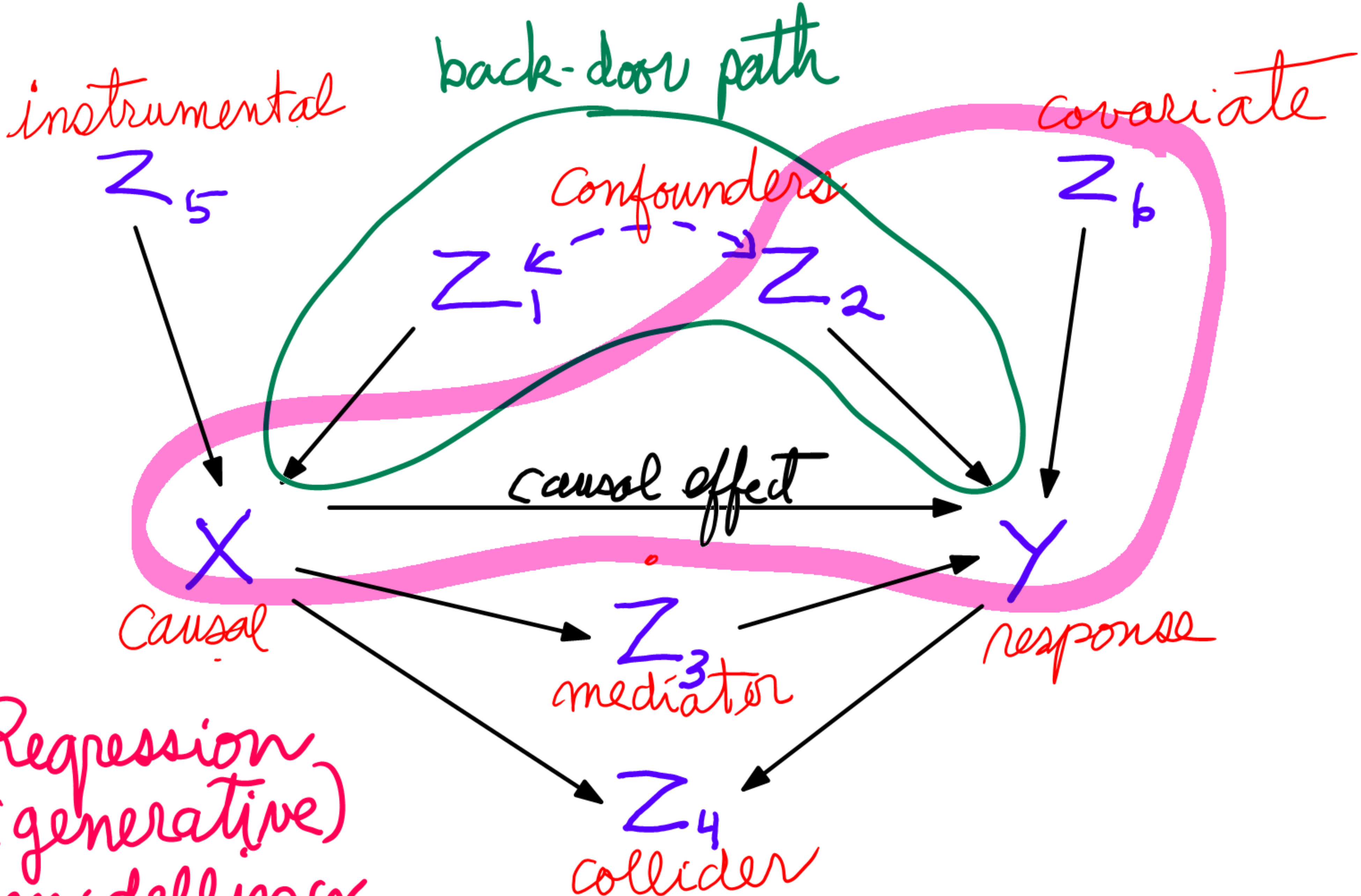
1) Random assignment of X


back-door path
instrumental
Z_5
covariate
Z_6
Confounders
Z_1
Z_2
Blocks
causal effect
X
Causal
Y
response
Z_3
mediator
Z_4
collider
1) Random assignment of X


back-door path
instrumental
covariate
confounders
OK
Z_5
Z_1
Z_2
Z_6
Blocks
causal effect
X
Y
causal
response
Z_3
mediator
Z_4
collider
NO
1) Random assignment of X



2) Regression (generative) modelling



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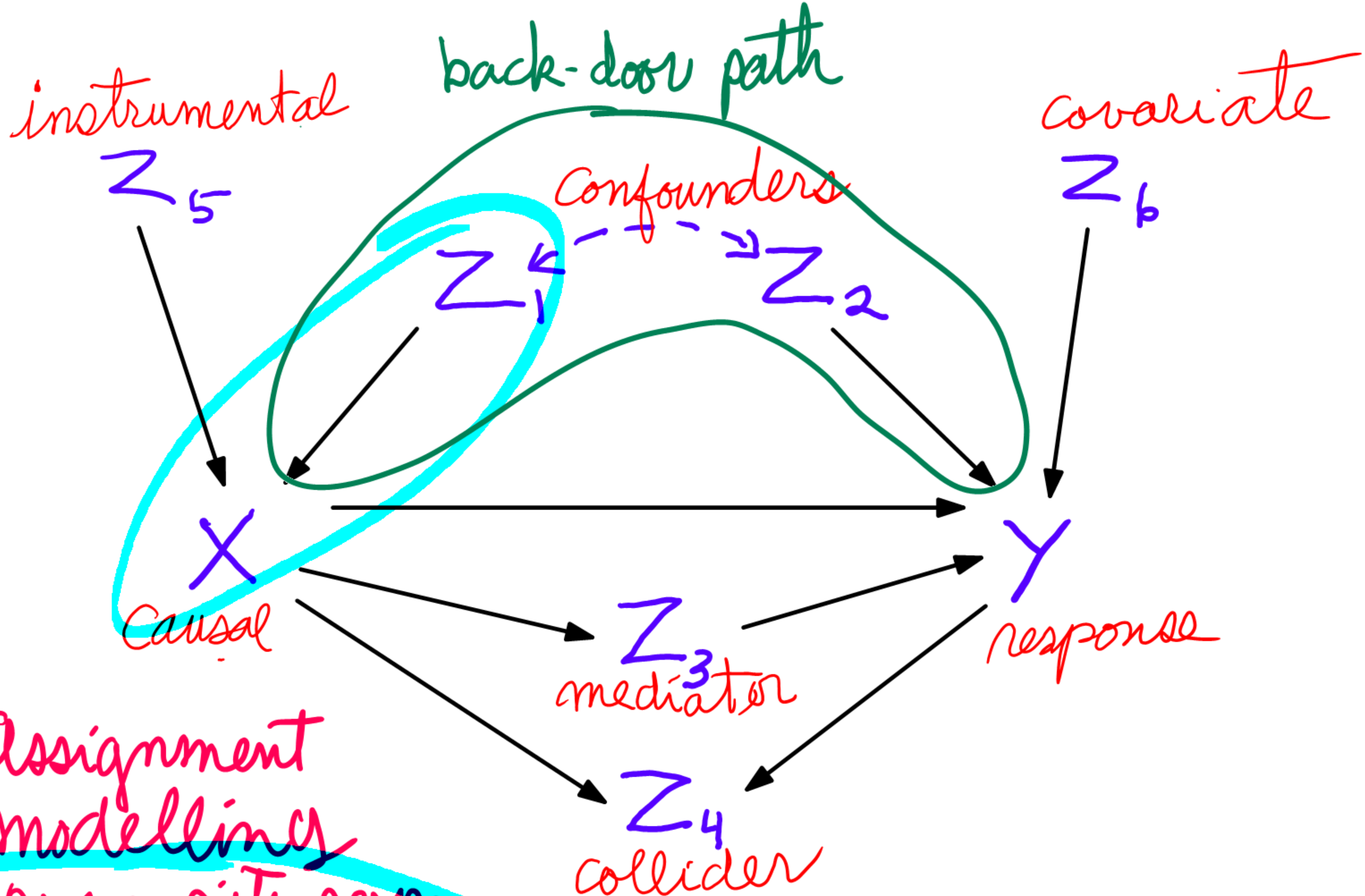
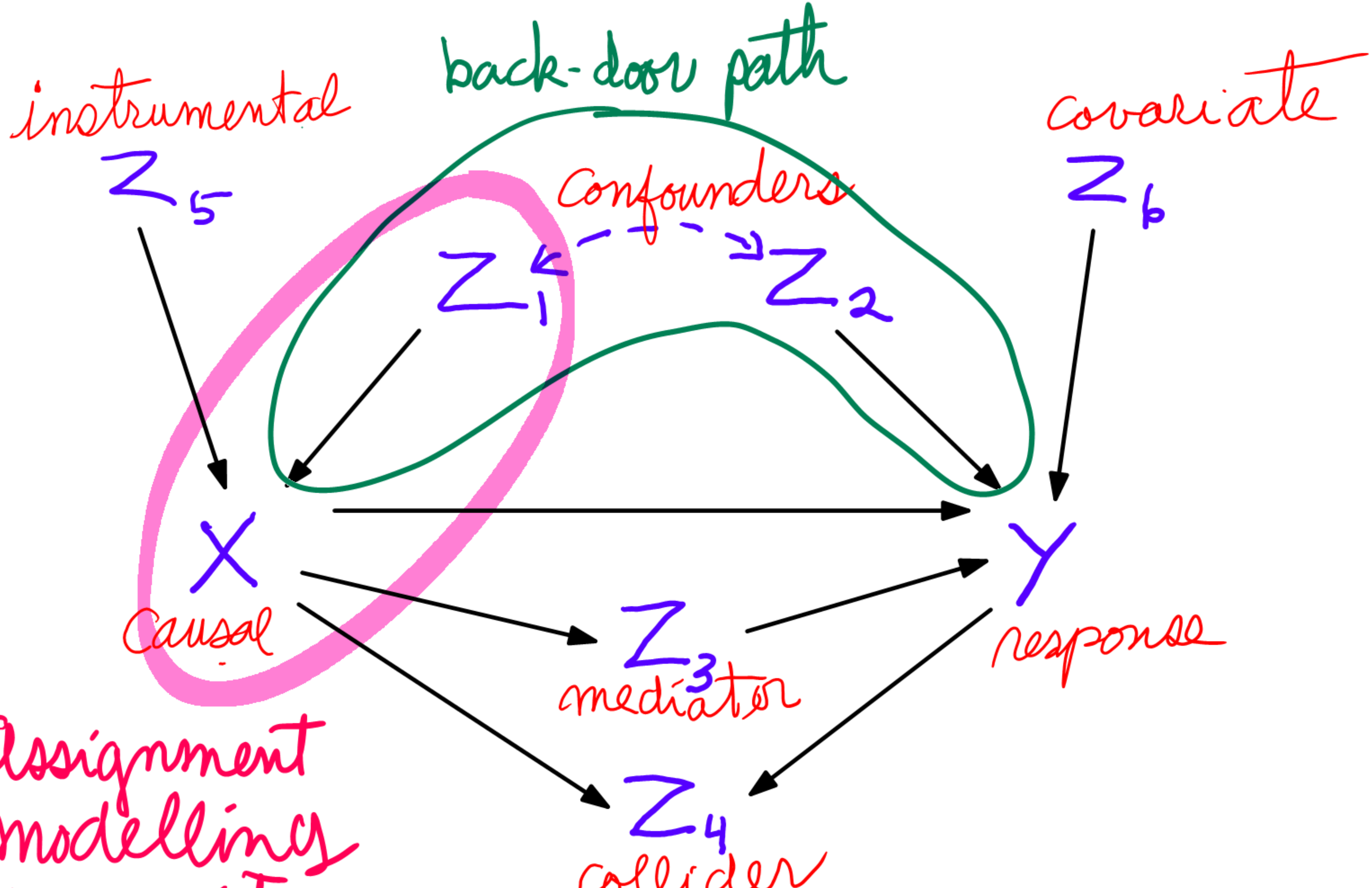


3) Assignment modelling
- propensity scores



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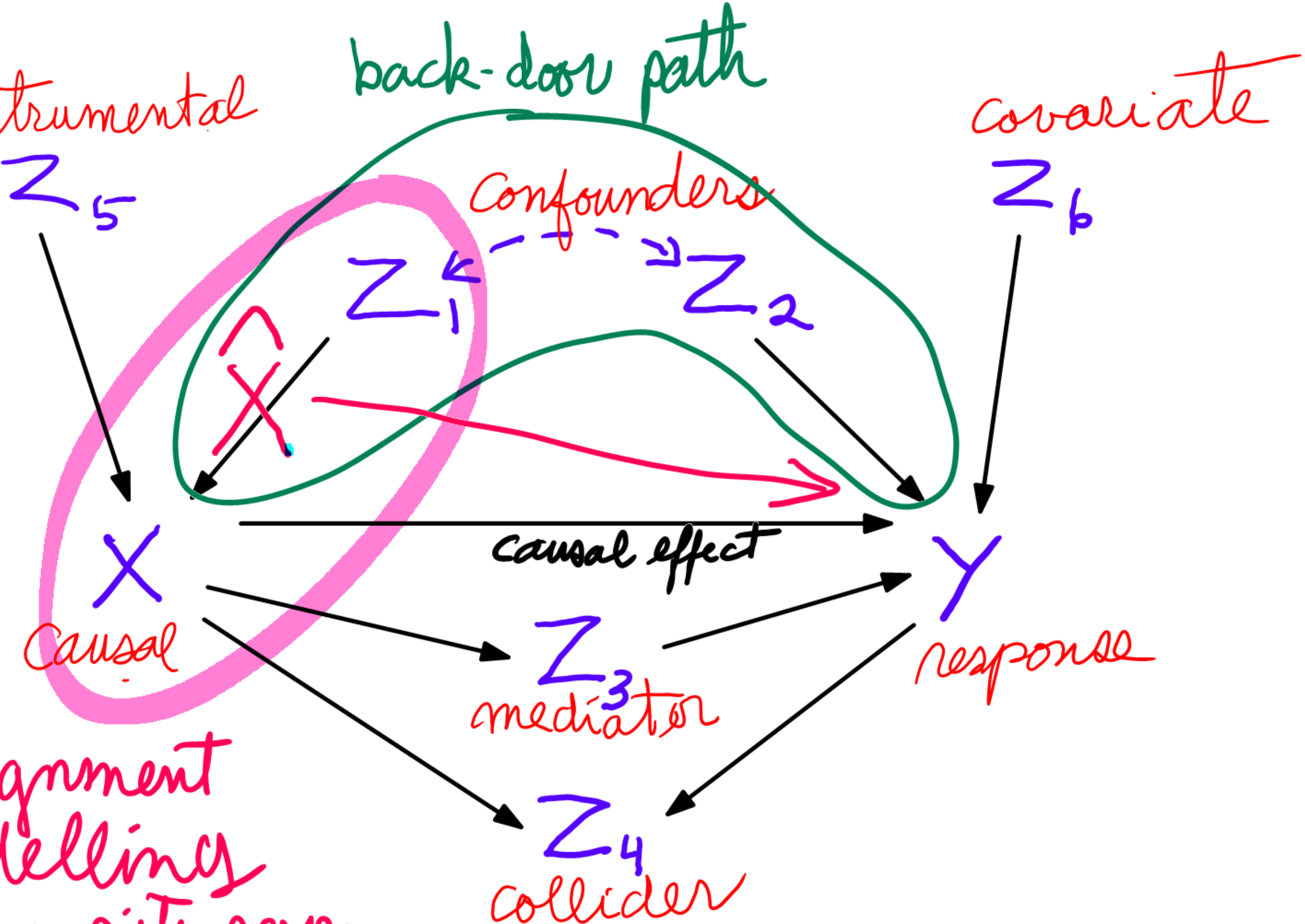

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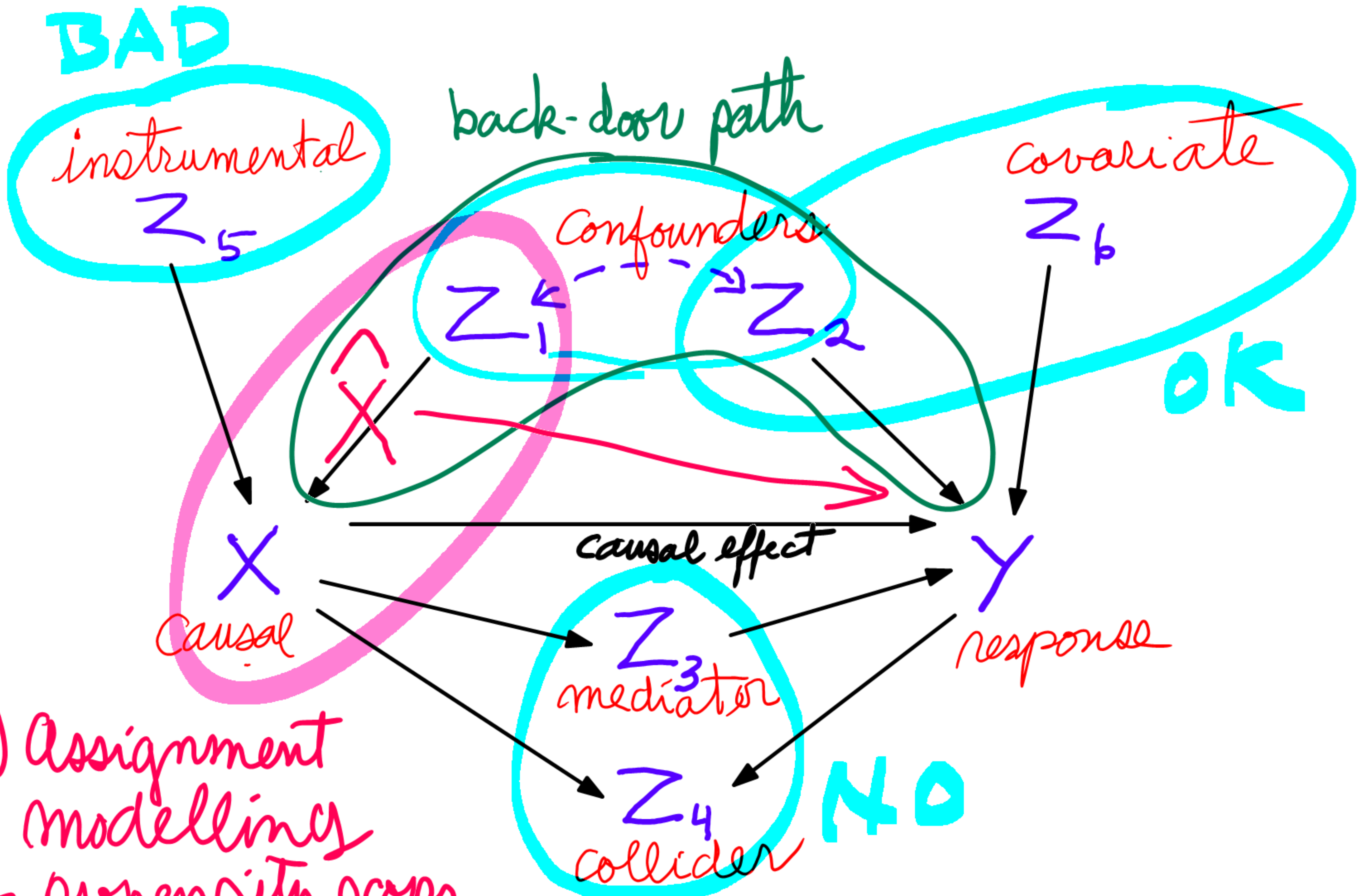

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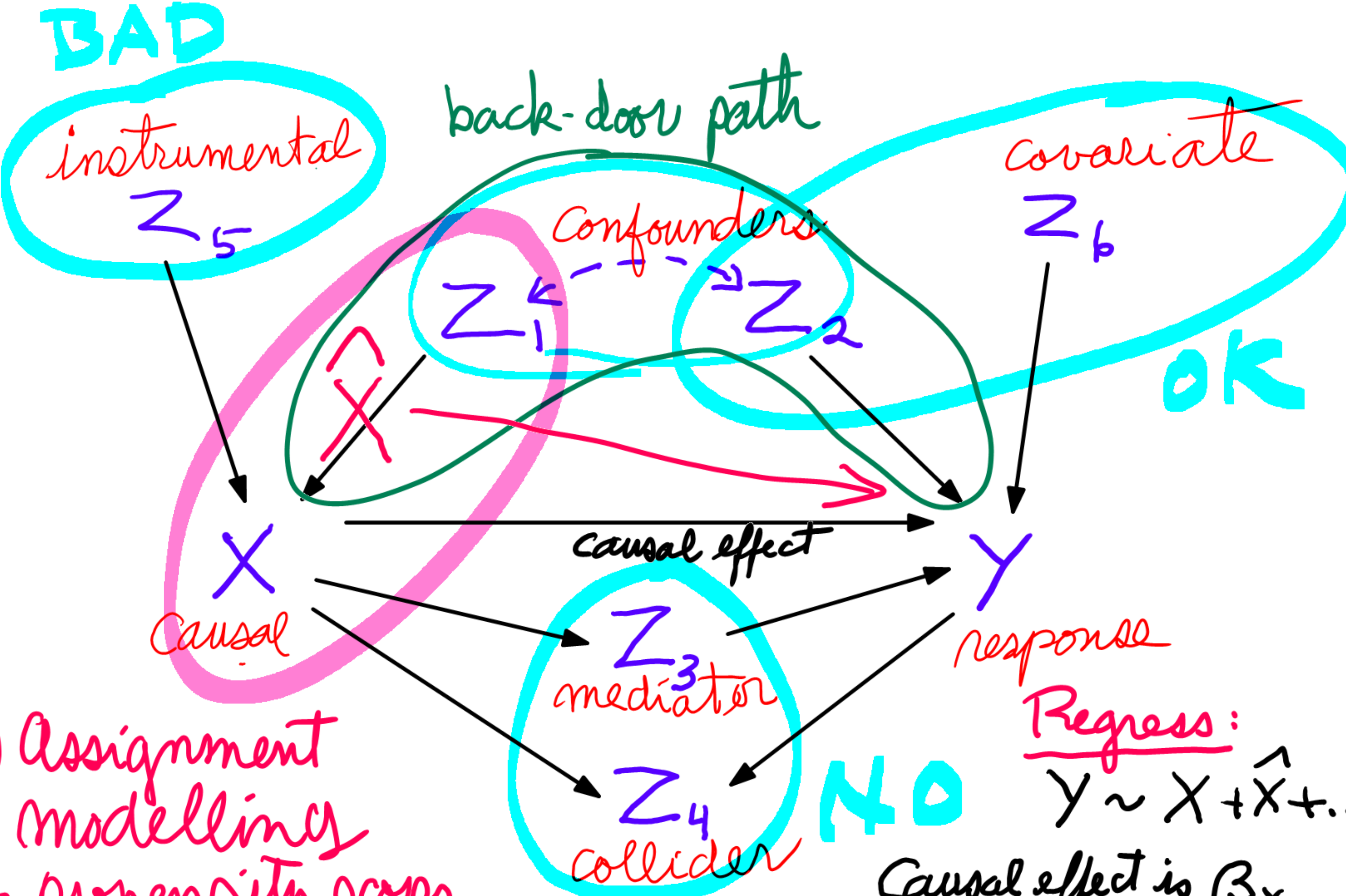
- propensity scores



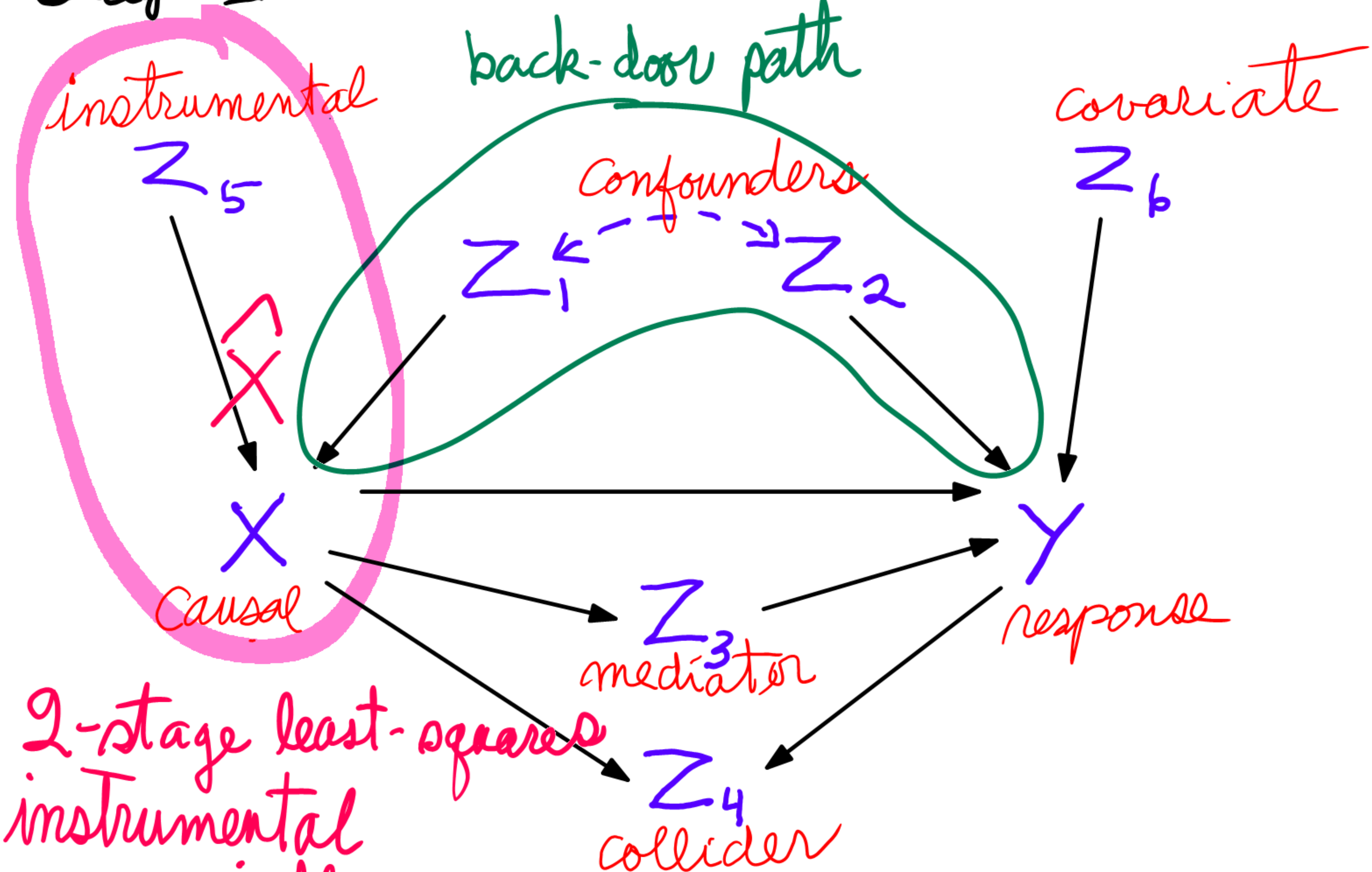
3) Assignment modelling
– propensity scores

Regress:

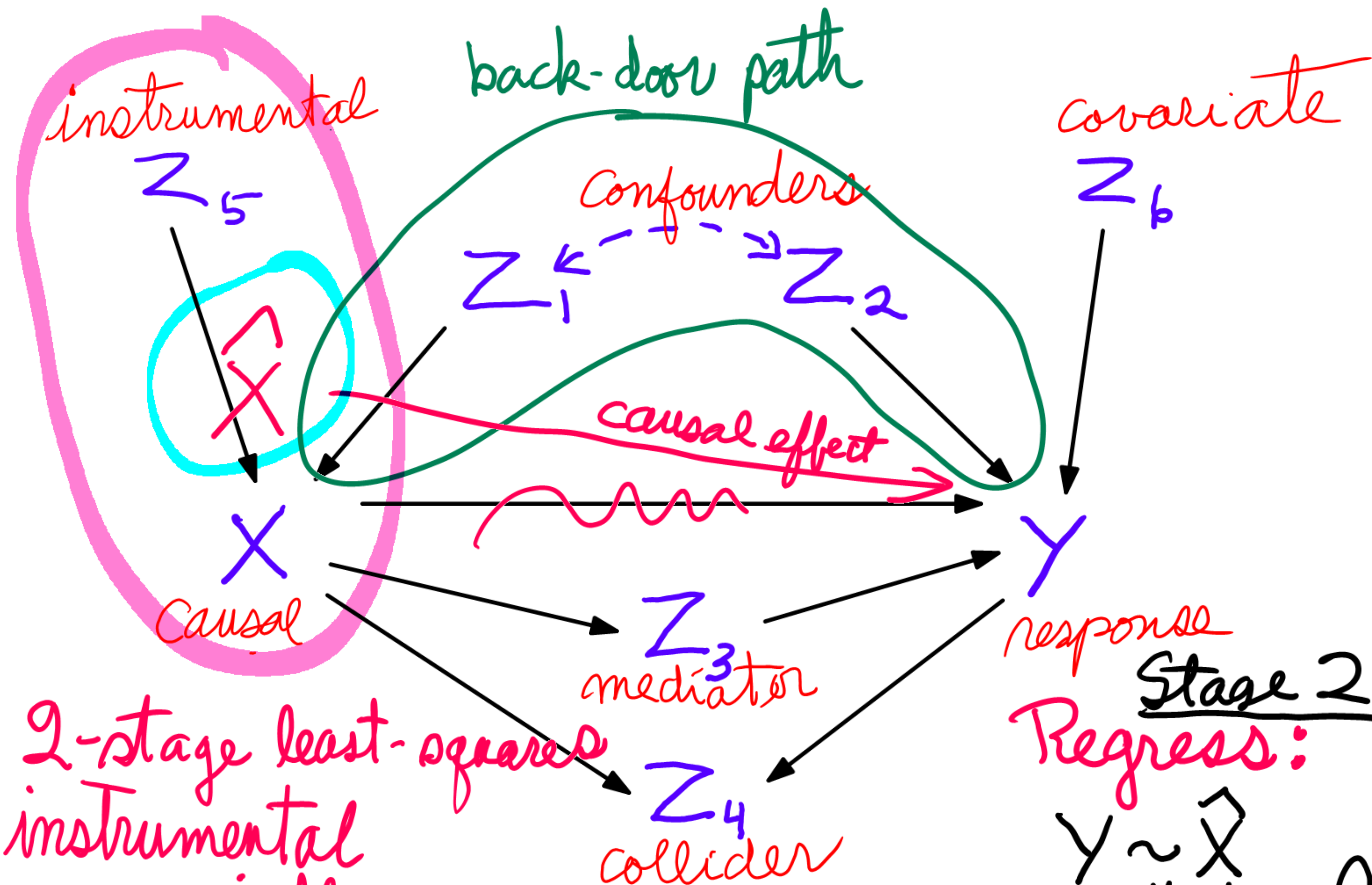
$Y \sim X + \hat{X} + \ldots$

Causal effect is β_X

Stage 1:



4) 2-stage least-squares instrumental variables



4) 2-stage least-squares instrumental variables

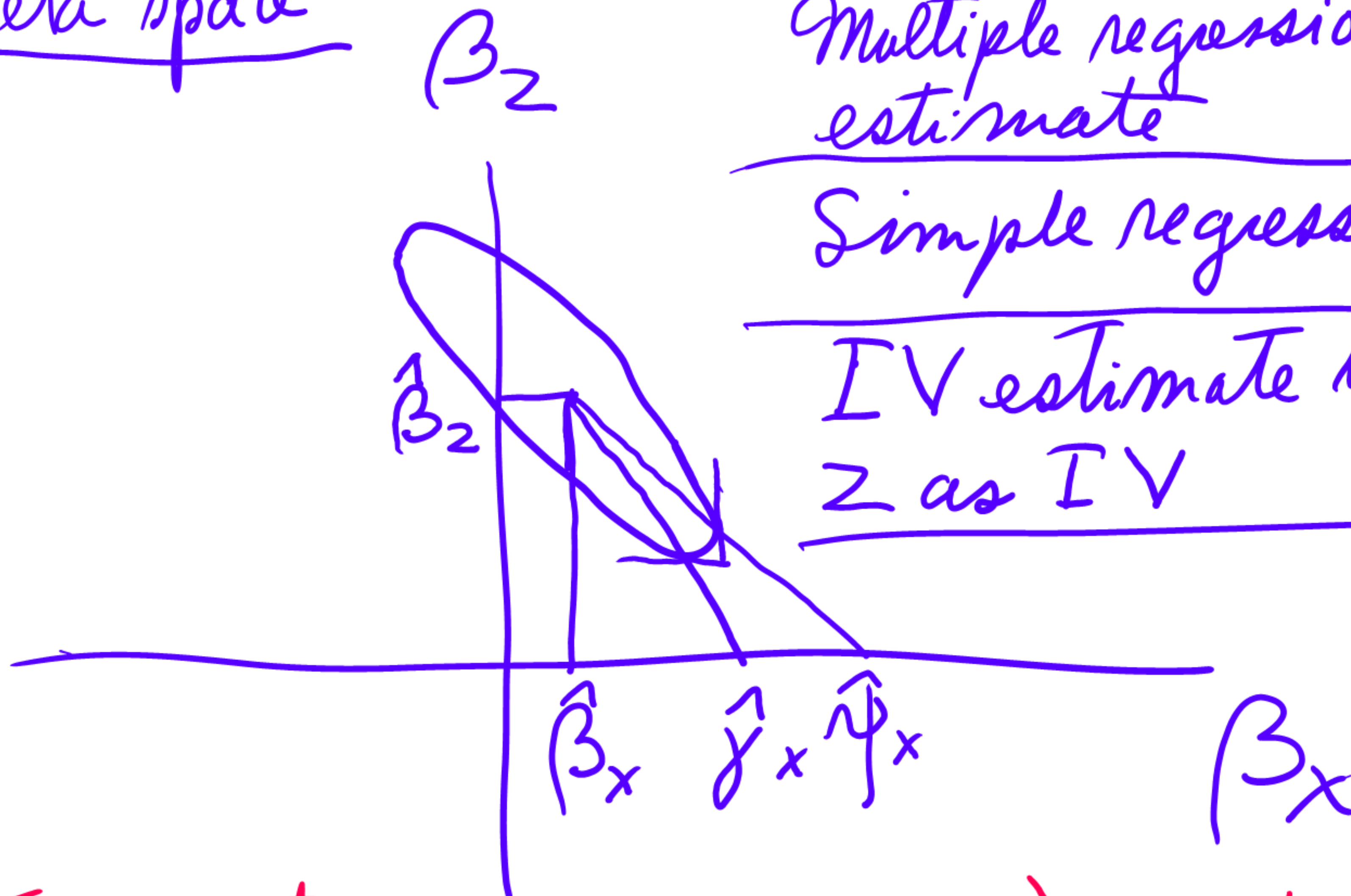
Stage 2

Regress:

$Y \sim \hat{X}$

Causal effect is $\beta_{\hat{X}}$

Beta space



Multiple regression estimate	$\hat{\beta}_x$
Simple regression	$\hat{\gamma}_x$
IV estimate with Z as IV	$\hat{\psi}_x$

For a strong IV, Corr(X, Z) close to 1 and exclusion restriction $\Rightarrow$ β_2 close to 0

So $\hat{\psi}$ close to $\hat{\gamma}_x$

Note that we can't test the assumption of "exclusion restriction" by looking at the coefficient of the instrumental variable I in the regression

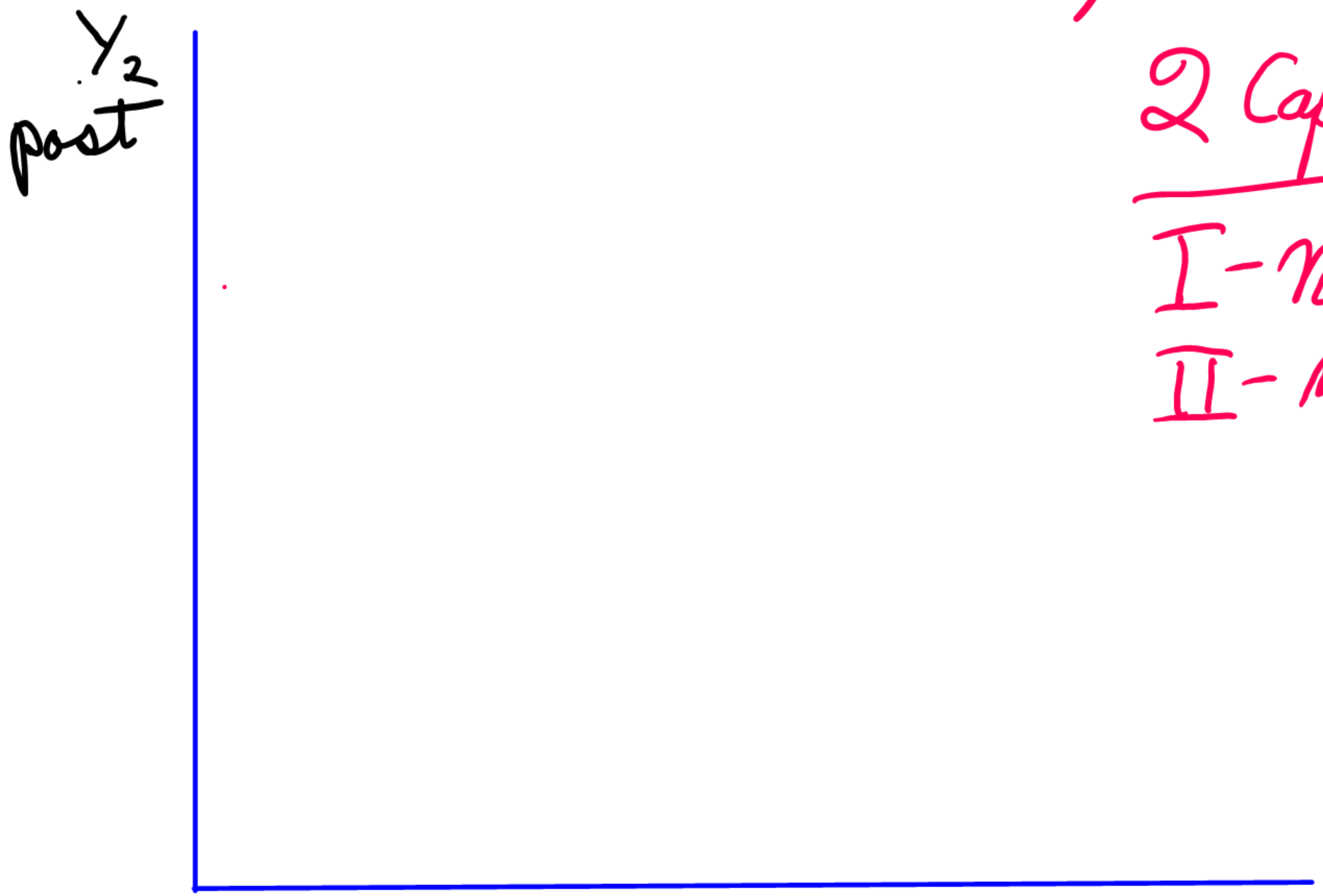
$$Y \sim I + X$$

since X is a collider if there is an omitted confounder C

$$\begin{tikzcd} I \arrow[dr] & & C \arrow[dl] \arrow[dr] & \\ & X \arrow[rr] & & Y \end{tikzcd}$$

and $\hat{\beta}_I$ should not be 0 even if I is a good instrument.

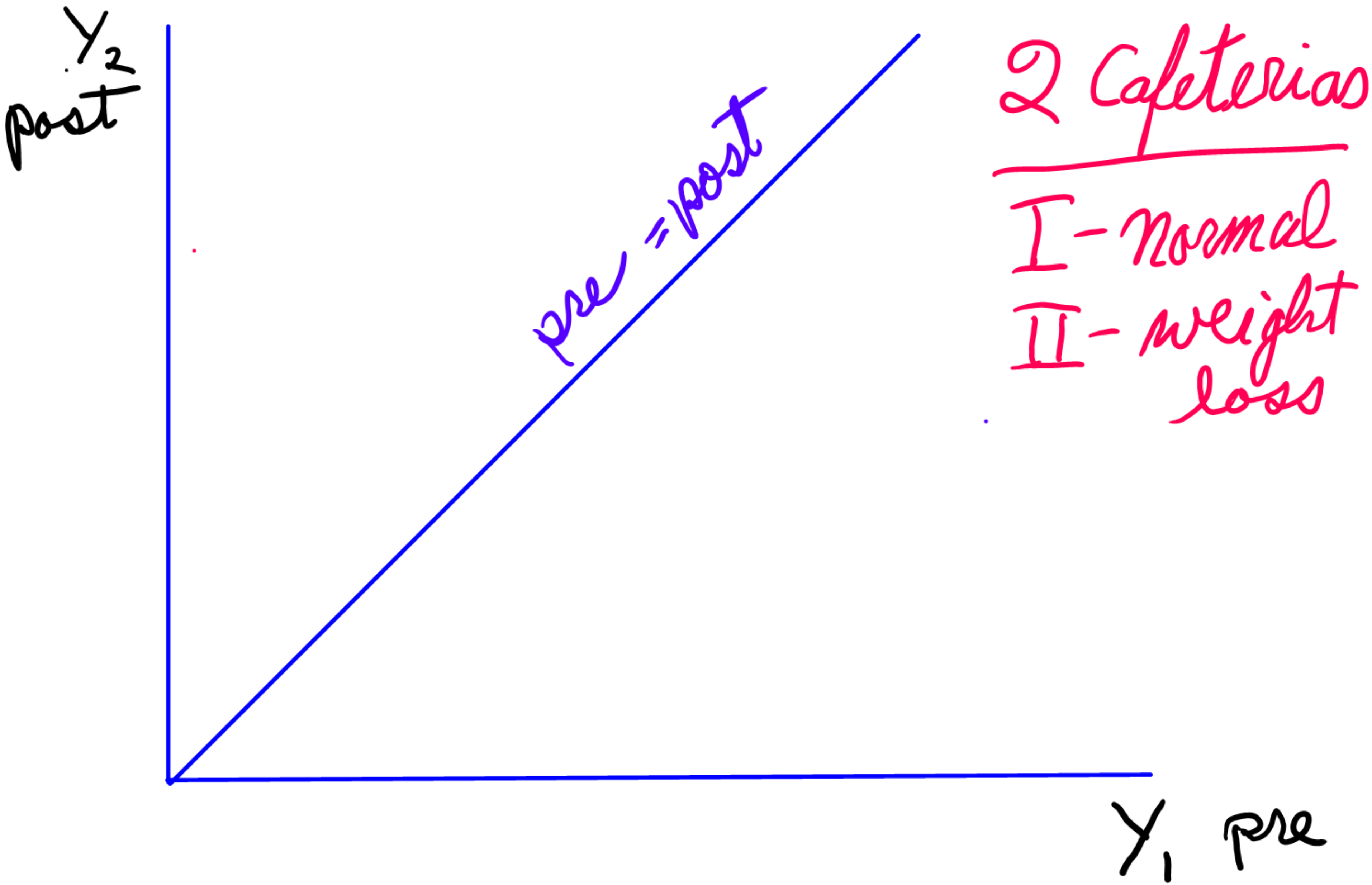
Lord's Paradox (Wainer version)



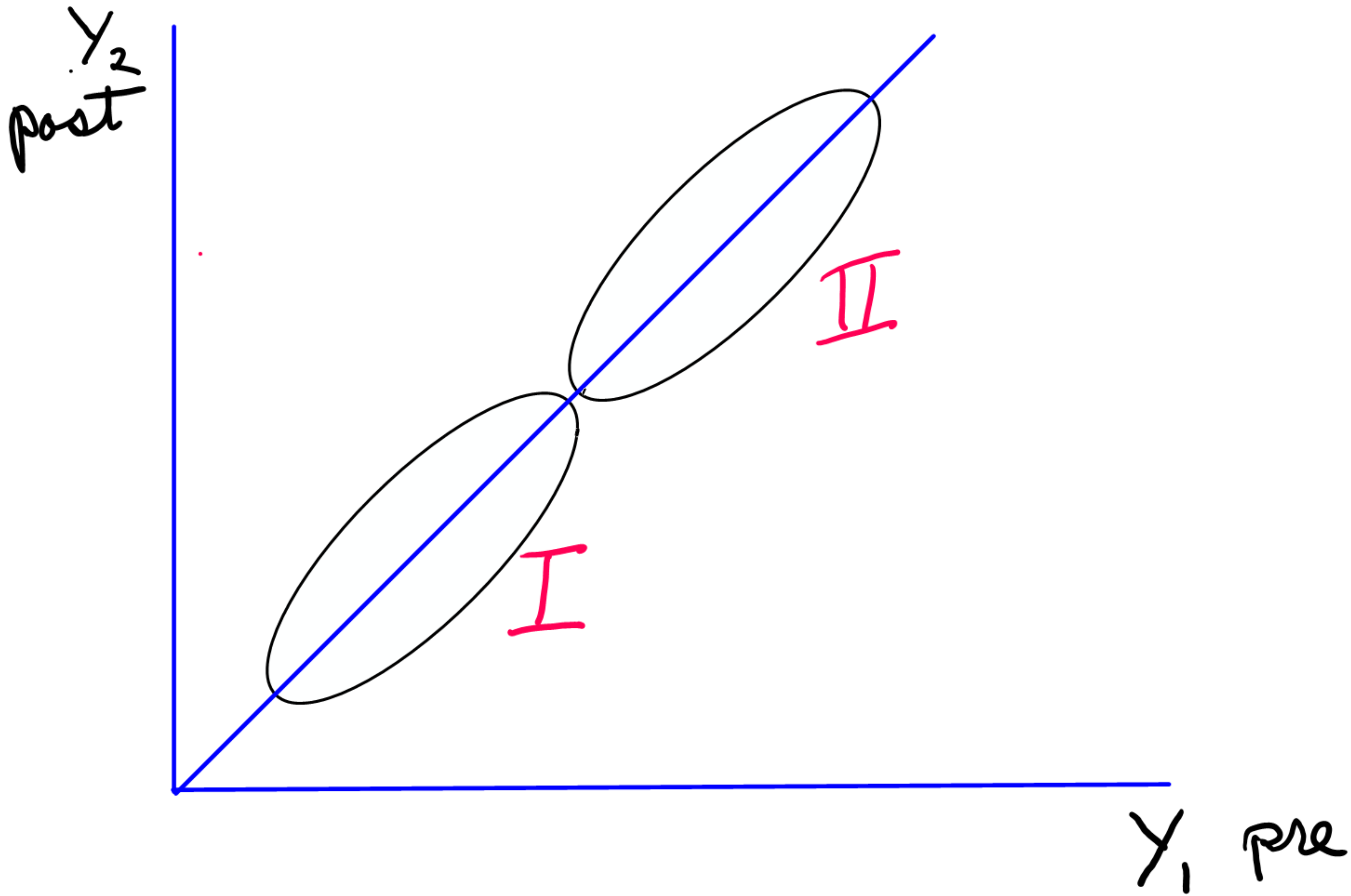
2 Cafeterias

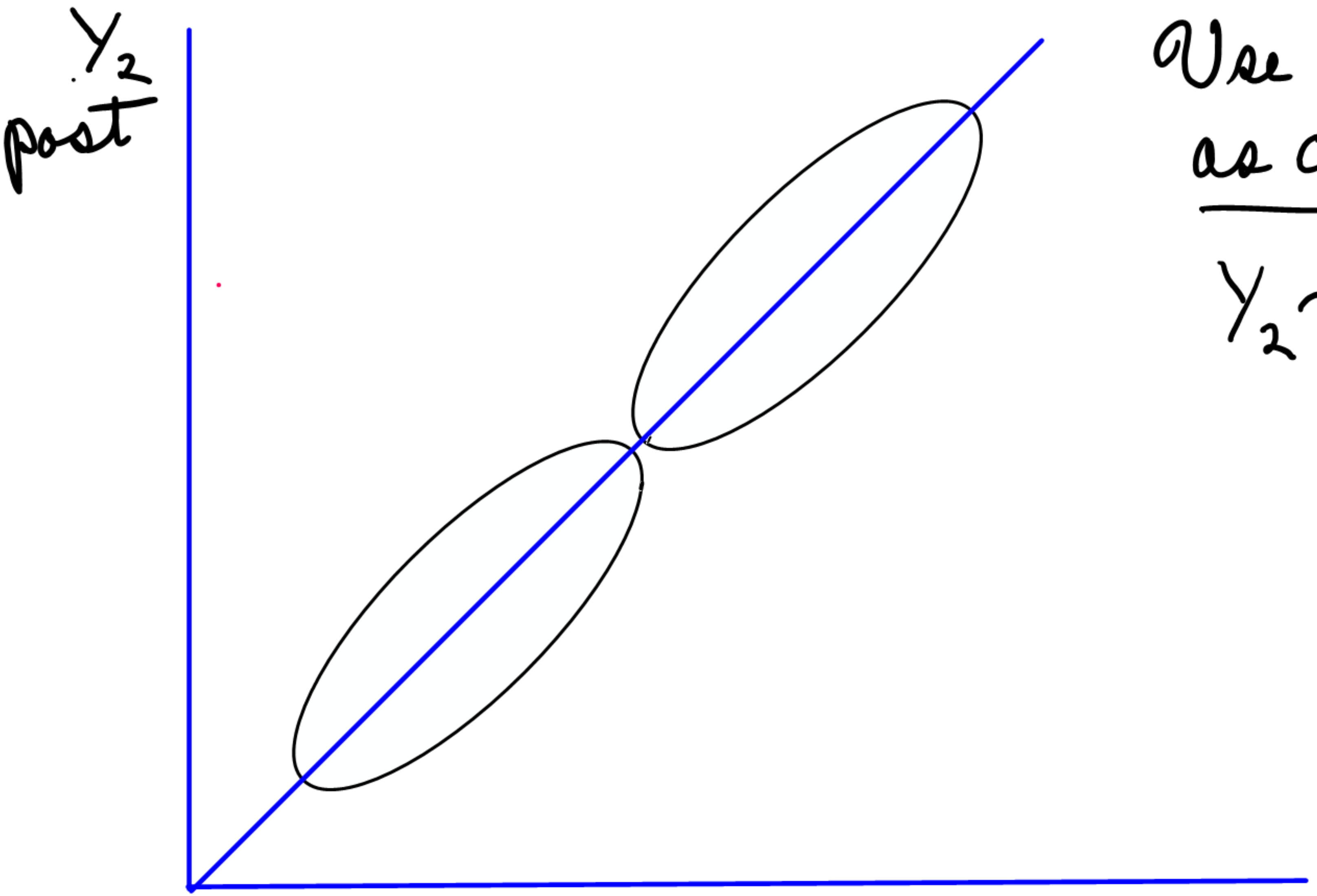
I - normal

II - weight loss



Y_2 post
pre = post
Y_1 pre
2 Cafeterias
I - normal
II - weight loss


Y_2
post
I
II
Y_1 pre

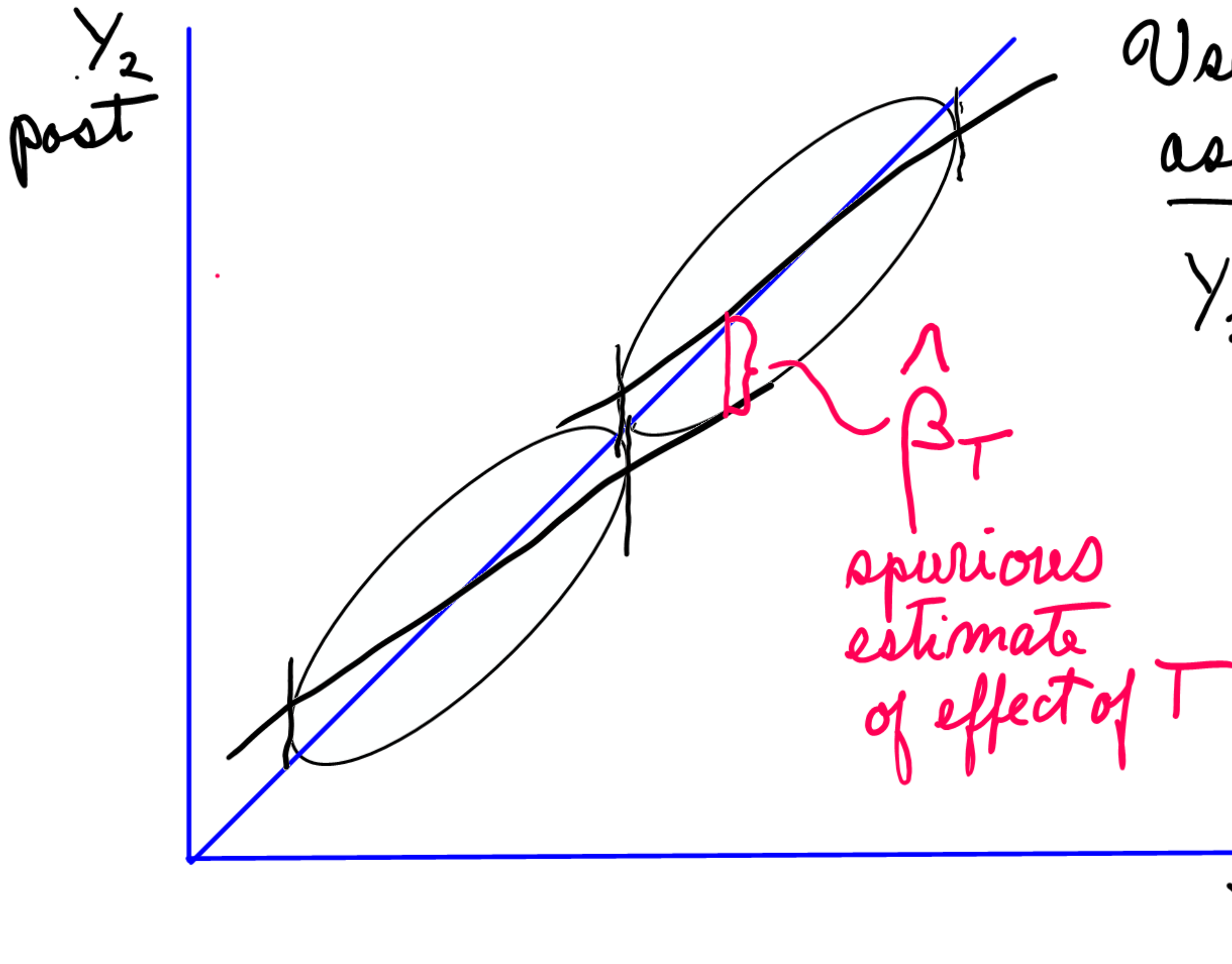


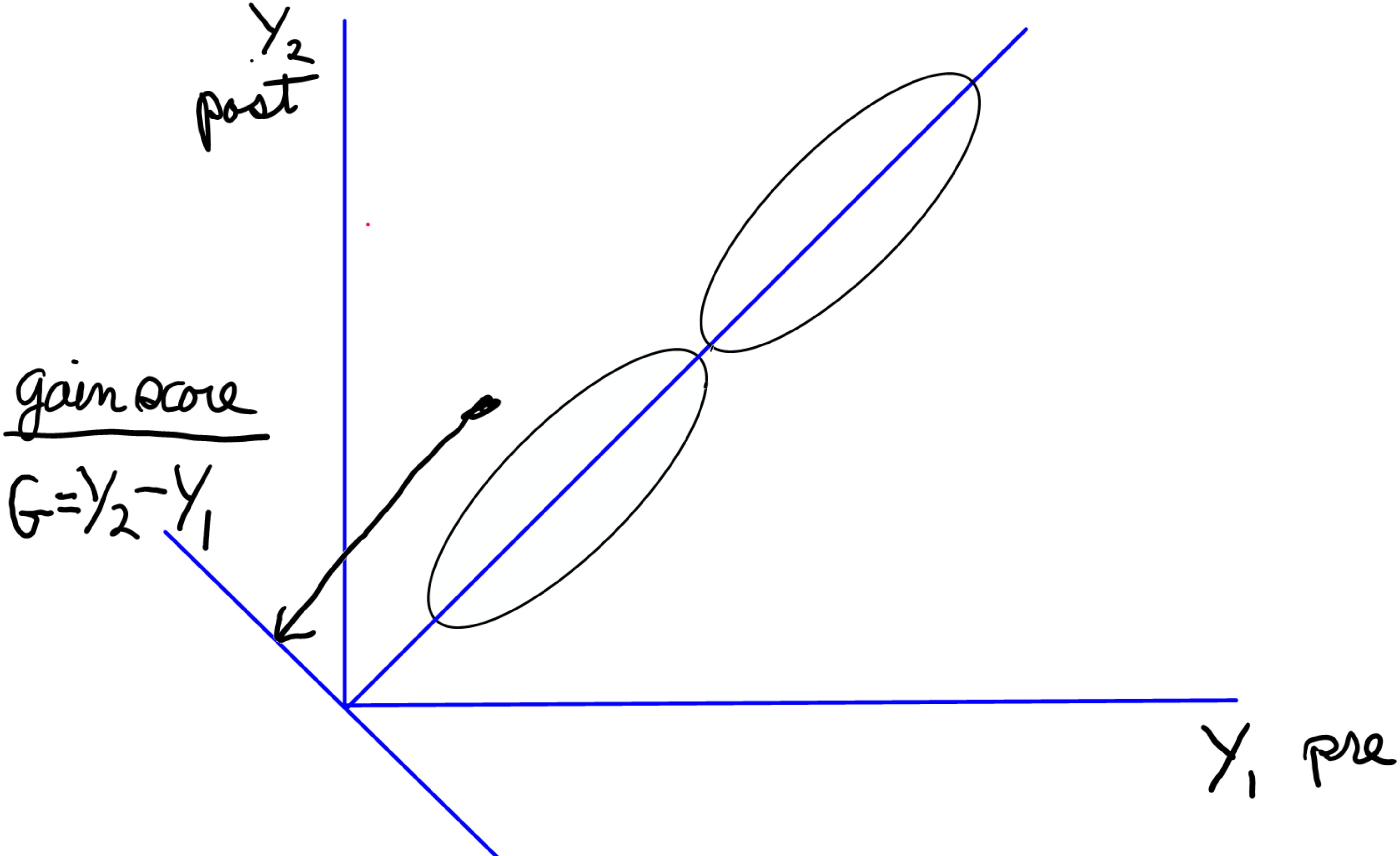
<u>Use pretest as covariate</u>

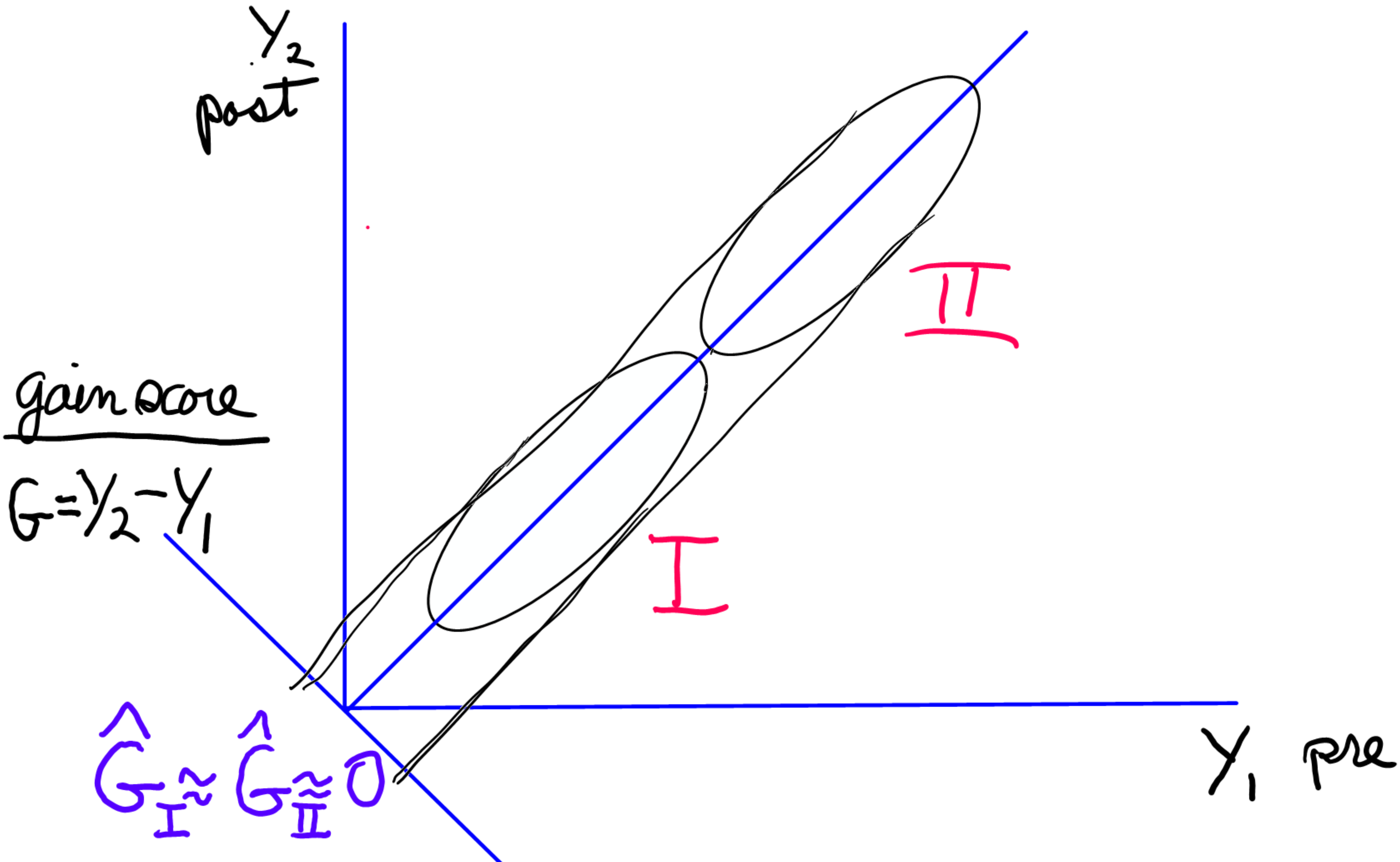
$Y_2 \sim T + Y_1$

Use pretest as covariate

$Y_2 \sim T + Y_1$

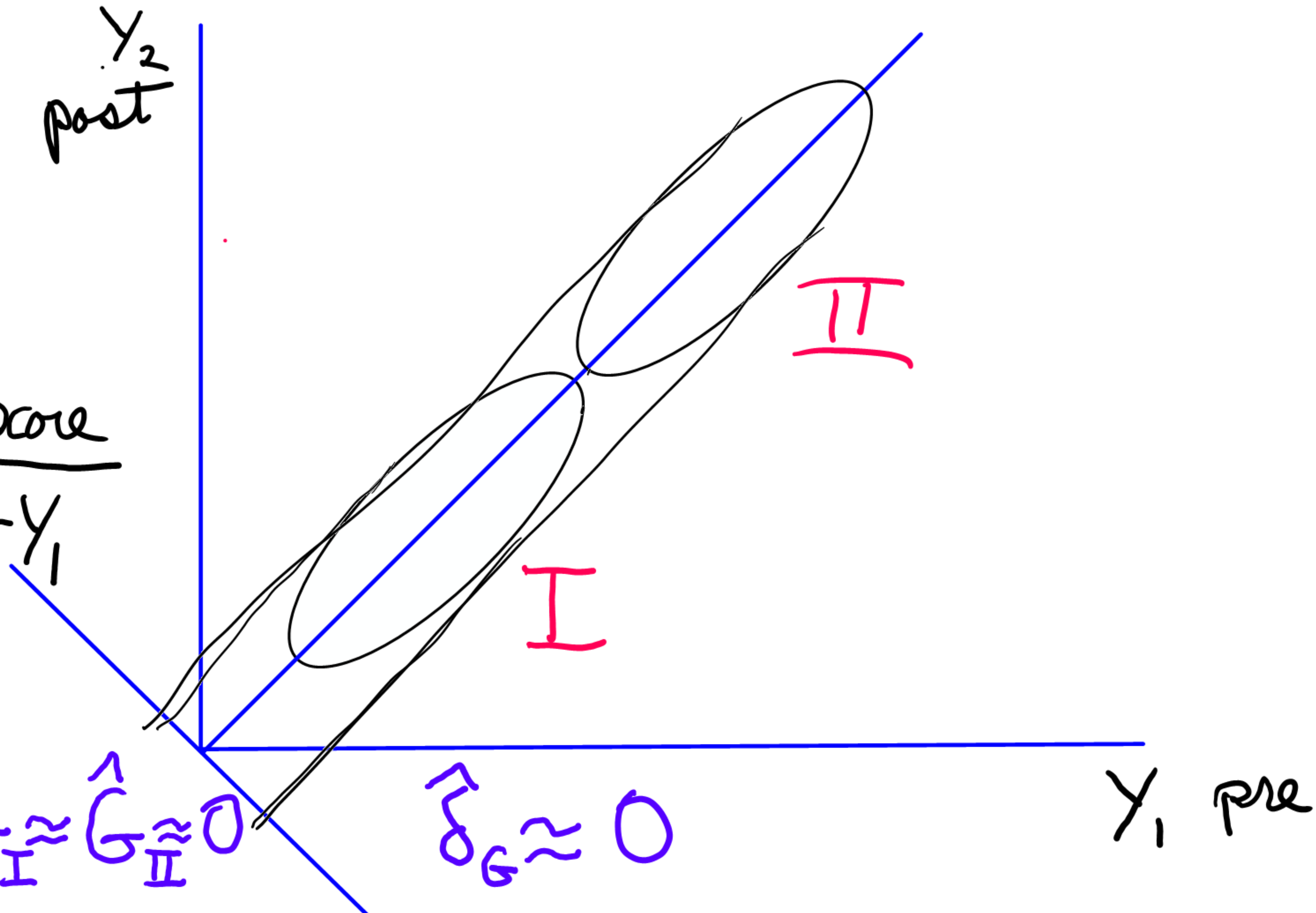



Y_2 post
gain score
$G = Y_2 - Y_1$
Y_1 pre


Y_2
post
gain score
$G = Y_2 - Y_1$
$\hat{G}_{I} \approx \hat{G}_{II} \approx 0$
I
II
Y_1 pre

gain score

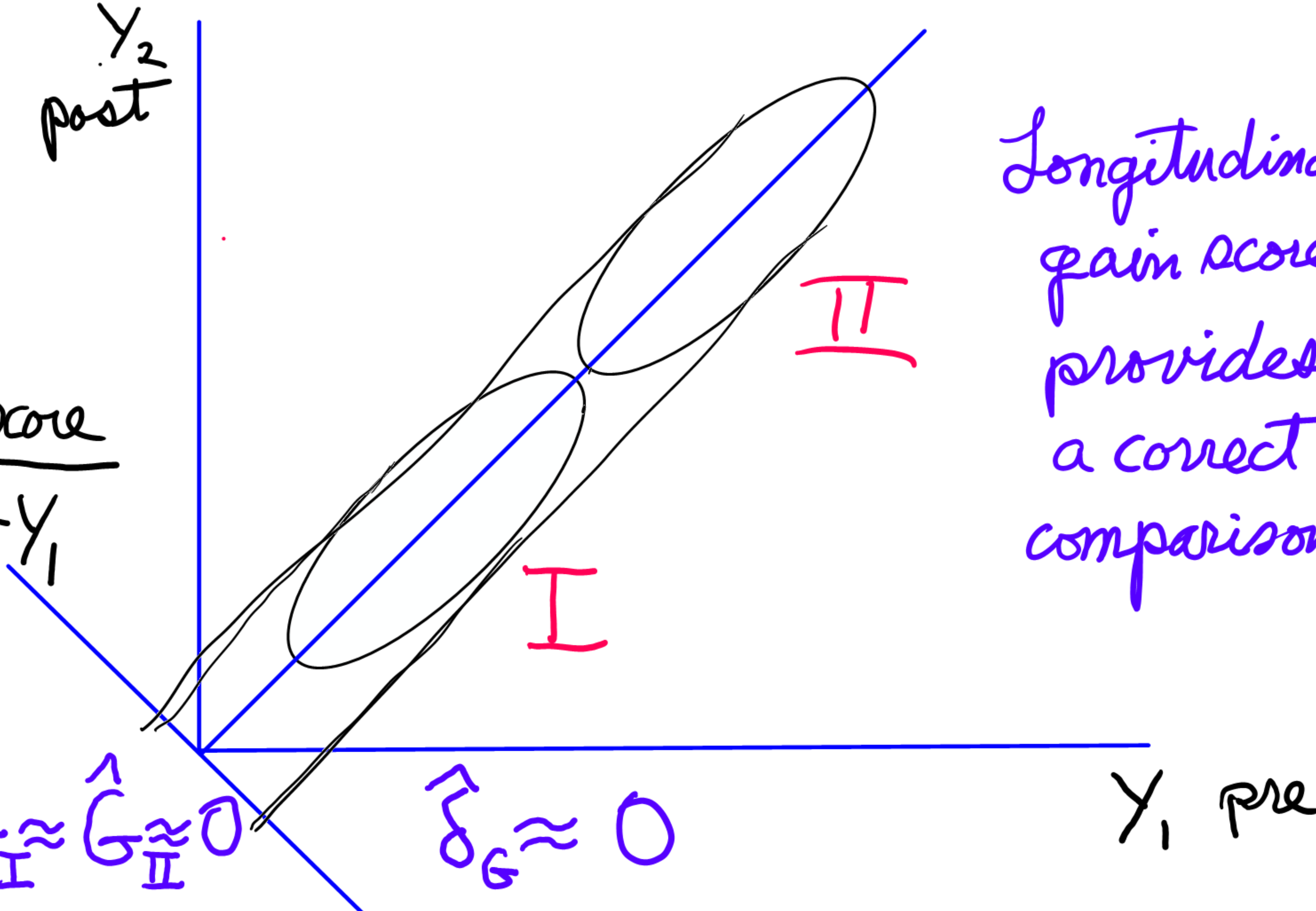
$G = Y_2 - Y_1$



$\hat{G}_{I} \approx \hat{G}_{II} \approx 0$ $\quad$ $\hat{\delta}_G \approx 0$

gain score

$G = Y_2 - Y_1$



$\hat{G}_{I} \approx \hat{G}_{II} \approx 0$ $\hat{\delta}_G \approx 0$

Longitudinal gain score provides a correct comparison

Conditions

- Same scale for Y_{pre} & Y_{post}
- No time-varying confounders

Within-subject effect adjusts for between-subject confounders whether measured or not.

Good model? $Y \sim X + Z_i + Z_j$

Want:

1) Unbiased · consistent

Block backdoors – NOT mediators & colliders

2) Low SE $= SD(Y_{res}) / SD(X_{res})$

Small $SD(Y_{res})$, Large $SD(X_{res})$

3) Honest SE

4) Robust   Propensity scores – focus on X

Use the AVP to compare models.

Using confounders close to Y

$$\left.\begin{array}{l}\downarrow SD(Y_{res}) \\ \uparrow SD(X_{res})\end{array}\right\} \downarrow SE(\hat{\beta}_T)$$

But may not have knowledge about structure of model for Y

Using confounders close to X

$$\left.\begin{array}{l}\uparrow SD(Y_{res}) \\ \downarrow SD(X_{res})\end{array}\right\} \uparrow SE(\hat{\beta}_T)$$

for X

But may have better understanding of assignment model

<u>Propensity score</u> methods focus on predicting X with $\hat{X}$

- no need to understand model for Y
- except to avoid mediators & colliders

Then regress Y on X and $\hat{X}$ (often grouped into intervals)

"<u>Doubly robust:</u>" throw in some Z's close to Y and covariates.