

$$X_i = \begin{bmatrix} 1 & -1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix} \quad Y \sim \beta + X$$

$$\text{random} \sim \mathbb{I}$$

$$Y_i = X_i \beta + \frac{1}{n} u_{0i} + \tilde{\varepsilon}_i$$

$$\text{Var}(u_{0i}) = G = [g_{00}]$$

$$\text{Var}(\tilde{\varepsilon}_i) = \sigma^2 I_{3 \times 3}$$

$$\text{Var}(Y_i) = \text{Var}\left(\frac{1}{n} u_{0i}\right) + \text{Var}(\tilde{\varepsilon}_i)$$

$$= \frac{1}{n} g_{00} \mathbb{1}' + \sigma^2 I$$

$$= g_{00} U + \sigma^2 I \quad U = \mathbb{1}\mathbb{1}'$$

$$\Sigma_i = \begin{bmatrix} \sigma^2 + g_{00} & g_{00} & g_{00} \\ & \sigma^2 + g_{00} & g_{00} \\ & & \sigma^2 + g_{00} \end{bmatrix}$$

$$\begin{matrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ & \sigma_{22} & \sigma_{23} \\ & & \sigma_{33} \end{matrix}$$

Random slope

$$Y_i = X_i \beta + X_i u_i + \varepsilon_i$$

$$\Sigma_i = X_i G X_i' + \sigma^2 I$$

$$= \begin{bmatrix} 1 & -1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} g_{00} & g_{01} \\ g_{10} & g_{11} \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ -1 & 0 & 1 \end{bmatrix} + \sigma^2 I$$

$$\begin{bmatrix} g_{00} - 2g_{01} + g_{11} & g_{11} - g_{10} & g_{00} - g_{11} \\ + \sigma^2 & & \\ & g_{00} & g_{00} + g_{10} \\ & & + \sigma^2 \\ & & g_{00} + 2g_{01} + g_{11} \\ & & + \sigma^2 \end{bmatrix}$$

