Suppose you have data on a continuous variable, Y, and two variables X and Z that are dichotomous with values 0 and 1.

The following table shows the values of $\bar{Y}$ for all the combinations of values of X and Z:

	X=0	X=1
Z = 0	8	6
Z = 1	3	2

The number of observations in each cell is:

	X=0	X=1
Z = 0	30	60
Z = 1	120	20

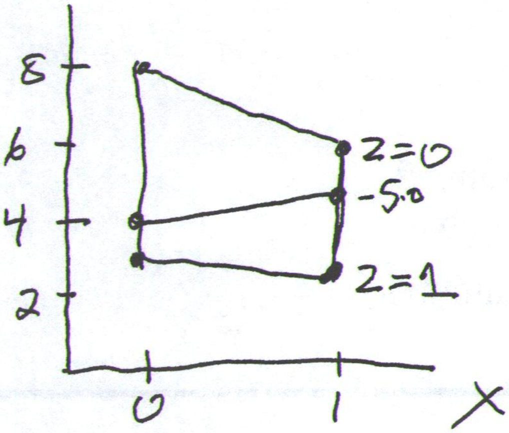
Question 1: (20 marks in 5 parts)

1. Draw the Paik-Agresti diagram for this data.
2. What is(are) the value(s) of the 'conditional effect(s) of X'?
3. What is the value of the 'marginal effect of X'?
4. If you were to fit the model `Y ~ X*Z` in R, i.e. the model

$$\hat{Y} = \hat{\beta}_0 + X\hat{\beta}_1 + Z\hat{\beta}_2 + XZ\hat{\beta}_3$$

   what would the values of the estimated regression coefficients be?
5. How would you get the 'conditional effects of X' from the regression coefficients? Express your answer in the form of a 'hypothesis matrix' multiplying the vector of fitted values, i.e. a matrix L so that your answer would have the form $L\hat{\boldsymbol{\beta}}$.

1)



2) ~~dif~~ $Z=0: \quad 6-8=-2$

$Z=1: \quad 2-3=-1$

3) $$\bar{Y}_{|X=1} = \frac{6\times 60 + 2\times 20}{60+20} = 5.0$$

$$\bar{Y}_{|X=0} = \frac{8\times 30 + 3\times 120}{30+120} = 4.0$$

marginal effect of X $= 5.0 - 4.0 = 1.0$

4) $$\hat{\beta}_0 = \hat{E}(Y \mid X=0, Z=0) = 8.0$$

$$\hat{\beta}_1 = \left.\frac{\partial \hat{E}(Y)}{\partial X}\right|_{Z=0} = -2.0$$

$$\hat{\beta}_2 = \left.\frac{\partial \hat{E}(Y)}{\partial Z}\right|_{X=0} = -5$$

$$\hat{\beta}_3 = \frac{\partial^2 \hat{E}(Y)}{\partial Z \partial X} = +1$$

5) $$L = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$